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STUDIES ON ANALYSIS, DESIGN AND CONTROL PLANNING OF WATER DISTRIBUTION NETWORKS

AKIHIKO UDO

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CONTENTS

CHAPTER 1 INTRODUCTION

1.1 Motivation of the Research

The average annual precipitation in Japan amounts to 1,800mm, which is fairly greater than the world average, 730mm. However, the annual precipitation per capita in Japan is about 6,000m^3, only one fifth of the world average. In addition, because of steep topography and short length of the rivers, much amount of the precipitation just runs away into the sea in a short time. Seasonal changes of the precipitation are also remarkable. In short, the natural condition is by no means favorable for efficient use of the water resources in Japan.

Uneven distributions of the precipitation and the population must also be noted. For typical instance, the amounts of the precipitation per capita in the districts of Kanto, Kinki and Northern Kyusyu, the most urbanized and industrialized districts in Japan, are less than one fifth of that in the Hokuriku districts. And then, in those urbanized and industrialized districts, the ratio of the total water demand to the amount of available water resources (the amount of the precipitation excluding evaporation) is at the level of 40% to 50% in a dry year /1.1/. As suggested by this high ratio, which is true more or less throughout the country, development of new water resources is now becoming more and more difficult and expensive.

On the contrary, demand for the water is increasing, and will increase in future too, due to needs for elevation of the living standard, development of socio-economic circumstances, etc. Therefore, it is now strongly desired to devise various means for efficient use of the limited water resources.

Basically, improvement of efficiency of water supply as well as economization of water at users is important for efficient use of the water resources. As practical means for the former, it is recommended (1) to reduce loss of water caused by overflow or invalid discharge from the open-channel distribution network by converting the network to closed conduit (pipe) one, and (2) to reduce leakage of water from the city-water distribution network by properly regulating the pressure in it /1.2/.

Moreover, huge amount of the social expense related to the water distribution network (hereafter abbreviated as WDN) should be noted. WDN constitutes the most downstream part of the whole water supply system. Though WDN is located at the most downstream part, its construction cost is not small: in fact, it amounts to more than 50% of that of the whole system /1.3/. In addition, electric power consumed for water distribution amounts to 1.12% of the total electric power generated in Japan /1.4/.

Thus, for efficient use of the water resources and for alleviation of the social expense, it is now becoming an important problem to develop practical methods for rationalizing design, management and control of WDN.

In the following, the present status of the existing analytical tools and optimization methods for design and control of WDN will be briefly summarized.

To begin with, the steady-state flow analysis (abbreviated as SSFA) is the very basic tool in working on WDN, and has been studied by many researchers. SSFA is ordinally done to obtain the inflow rates from sources (reservoirs) and the water pressures at demand terminals, prescribed with the water levels in reservoirs, the outflow rates from terminals (consumptions), and all the characteristics of pipes, pumps and valves. Hereafter, we call such SSFA the basic SSFA.

So far, as a matter of fact, most problems of design and control of WDN have been approached by iterating SSFA. That is to say, a designer confronted with a design problem, first, determines temporal values of design variables such as pipe diameters, based on his knowledge obtained through experience and/or some authorized criteria, and next, iterates SSFA and modification of the design variables until pressure conditions and some other requirements are satisfied. Control problems also have been solved likewise: an operator determines the setting of pumps and/or valves by iterating SSFA and modification of the setting.

Of course, a variety of methods for optimizing design and/or control have been proposed so far, and some of which, especially those for branching networks, have been put in practical use. For large-scale looped networks, however, most of those methods have some weakness from practical viewpoints. This is mainly due to the following reasons.

(1) Problems of WDN are nonlinear due to certain nonlinear physical laws governing the network.

(2) Various factors such as cost, supply reliability, water quality, must be taken into account for optimization, and some of them are difficult to be quantitatively evaluated and conflict with each other.

Nowadays WDN's are getting larger in size and more complicated in structure as a result of growth of city areas and/or integration of networks. Accordingly, applicability of the existing methods is getting poor.

Based upon the above observations, this dissertation is devoted to the development of some practical methods of design and control of WDN which are endowed with excellent applicability to large-scale networks. Much effort is also directed to a generalization of formulation and establishment of fast solution methods of SSFA, because a number of SSFA are frequently made for a variety of purposes in the business of water distribution, including for solving some classes of optimization problems.

1.2 Description of the Contents

The dissertation consists of six chapters including this introductory chapter.

Chapter 2 is concerned with the fundamentals of mathematical models of WDN's. Although the models in the chapter still belong to the class of traditional ones, they are presented in much more refined and generalized manners as possible. Hence they can effectively be used for formulating a variety of problems, especially steady-state problems such as discussed in this dissertation.

Chapter 3 develops a comprehensive study on SSFA. It begins with a systematic investigation on the basic SSFA to establish the most efficient method for fast solution of large-scale networks: methods of formulation, methods of fast solution, and further desirable linked usage among them are examined.

Generalization of SSFA is also the main purpose of this chapter. In several of real applications to design and/or control problems of WDN, the characteristics of hydraulic elements, such as pipe diameters, pump heads, etc. can not be known a priori; while, the inflow rates from the existing reservoirs as well as their water levels are measured and then known quantities. The generalized SSFA developed in this chapter gives more freedom than the basic one in choosing quantities to be prescribed and to be calculated, which is favorable for study of a wide class of real design and/or control problems. The most efficient linkage of methods for formulation and solution of the generalized SSFA is also proposed.

Chapter 4 presents new methods for modeling and design of the irrigation-water distribution network (abbreviated as IWDN). First, a "link outflow model" is proposed. A major feature of IWDN is that water is usually withdrawn almost uniformly along terminal links (pipes). The proposed model reflects this feature, and makes a more precise analysis and design of IWDN possible than before. As the model is a revision and a generalization of the traditional "nodal outflow model" used for the city-water distribution network (CWDN), it is also applicable effectively for CWDN.

Second, a method of design is proposed which has prominent features such as follows; (1) it enables us to consider explicitly the trade-off between enhancement of the supply reliability and reduction of the construction cost, (2) it is applicable to the multi-source network as well as to the single source network, (3) it requires solution of merely a set of simultaneous algebraic equations and a linear or a separable programming problem, then it is quite feasible even for large-scale networks which have about 1,000 nodes, 1,500 links and/or 300 loops. Further, it is applicable to CWDN in the like manner.

Last, the proposed methods are applied to a variety of networks and their feasibility is validated.

Chapter 5 is concerned with some basic problems for planning of uniform pressure control. The uniform-pressure-control planning problem is a generic name of problems to find out a proper way for operations of pumps and valves so as to set pressures in the network as low as and as uniform as possible, prescribed with water levels in reservoirs and consumptions at all demand nodes. The prominent features of the proposed method are summarized as follows: (1) it enables us to find out desirable locations of pumps and/or valves as well as their operation schemes, and (2) the procedure of solution is composed of iterative solutions of linear programs and, therefore, the method has an excellent applicability to large-scale networks.

The last chapter, Chapter 6, describes general conclusions as well as some open problems for further study.

Finally, it is noted that the problems treated in this dissertation are solely classified into steady-state ones. Status changes in WDN are relatively slow under normal situations. Therefore, discussions on representative values for some period of time, for example, one or half an hour, are quite meaningful, from a practical viewpoint, and give us much knowledge with relatively small endeavor.

CHAPTER 2 MATHEMATICAL MODELS OF WATER DISTRIBUTION NETWORKS

2.1 Introduction

This chapter is concerned with the fundamentals of mathematical models of water distribution networks (briefly WDN's), especially city-water distribution networks (CWDN's). Those topics such as traditional way of simplification in developing the mathematical model of a network, graph theoretical treatment of the model, principal variables appearing in the model, characteristics of hydraulic elements, and physical laws governing the network, are discussed.

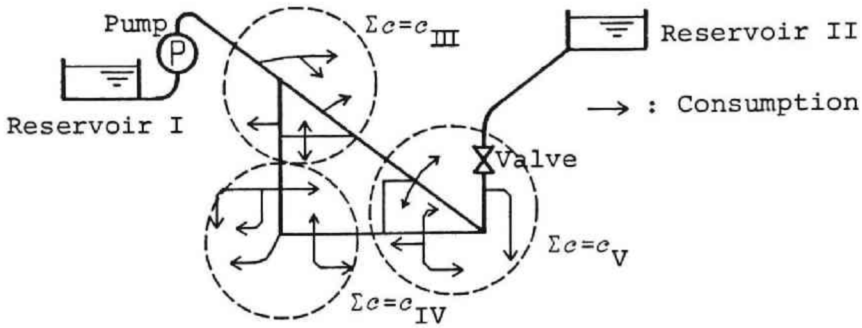
Although the models in this chapter still belong to the class of traditional ones, they are presented in much more refined and generalized manners as possible. Hence they can effectively be used for formulating a variety of problems, especially steady-state problems such as discussed in this dissertation.

A new method of modeling which enables us a more precise analysis and design of WDN's will be developed in Chapter 4.

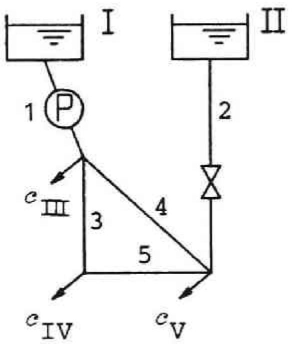
2.2 Network Model and Its Graph

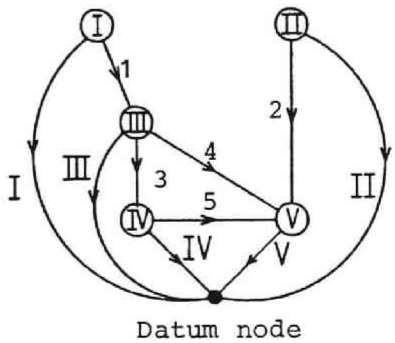
Figure 2.1(a) illustrates a simple CWDN. Purified water is conveyed from reservoirs to all consumers through pipes, and its flow is often regulated by pumps and/or valves on the way.

In studying the CWDN, it is traditionally simplified as shown in Fig. 2.1(b). That is, only main pipes whose diameters are 75mm (or 100mm) and above are taken into consideration. Consumptions withdrawn from the network are properly lumped and



(a) Illustrative example of a CWDN





(b) Simplified network (c) Linear graph G of (b)

Fig. 2.1. Model of a CWDN and its graph.

made associated with the nodes where those main pipes are interconnected. Let us call such a model a "nodal outflow model (N-model)" hereafter.

In studying steady-state problems graph theoretically, it is convenient to treat the network as if it were closed one. Again by way of example, the network of Fig. 2.1(b) is represented by the graph G of Fig. 2.1(c). In G, connections of hydraulic elements are denoted by nodes, and hydraulic elements by line segments connecting nodes. Hereafter we call these line segments "links". It should be noted that the reservoirs, the consumptions as well as the pipes, the pumps and the valves are represented by links, and that all the links of the reservoirs and the consumptions are interconnected at an imaginary datum node. Let us call the links of the reservoirs and the consumptions "in/outflow links", and the other links "conduit-element links", considering their characters. Let the number of in/outflow links be n, and that of conduit-element links be m.

2.3 Principal Variables

The principal variables for describing WDN consist of the head (pressure) and the flow rate which correspond to the voltage and the current in the electrical network (EN), respectively.

If an open end vertical pipe of sufficient length is connected to the network at a point, water rises in the pipe to a level. The head (total head) at the point is the height of the top surface of the water scaled from the level of the datum node, which is usually assumed on the sea level. By the way, the water pressure really available at the point is the length of the water column itself, i.e., the head excluding the ground level of the point, and this is called the effective head. The difference of heads at the both ends of a link is called the (link) head difference. Since the head at the datum node is zero, the head difference of an in/outflow link is nothing but the head at the node which is connected to the datum node through the link.

We denote a set of the head differences of all links by a vector $H=(h_i)$, and a set of the flow rates of all links by a vector $Q=(q_i)$. The sizes of H and Q are both $(m+n)$.

The MKS units are used throughout this dissertation, hence the units of H and Q are [m] and [m^3/s], respectively.

2.4 Characteristics of Hydraulic Elements /2.1/-/2.2/

2.4.1 Pipes

Some empirical formulae are popularly used for describing the relationship between the head difference (loss) h_i and the flow rate q_i of pipe i. Those are the Hazen-Williams equation, the Manning equation, the Watson equation, and so on, and they are properly used depending on status of the inner walls of pipes. Whichever equation is used, however, there is no essential difference in mathematical treatment. Hence the following formula of Hazen-Williams, which may be most popular, will be used:

$$h_i = r_i q_i |q_i|^{0.85},$$
$$r_i \triangleq 10.666 CH_i^{-1.85} d_i^{-4.87} l_i \quad \text{(resistance factor)} \qquad (2.1)$$

where

CH_i: smoothness coefficient of pipe i [-]

d_i: diameter of pipe i [m]

l_i: length of pipe i [m]

Solving Eq. (2.1) with respect to q_i gives

$$q_i = y_i h_i |h_i|^{-\beta} = y_i (h_{io} - h_{id}) |h_{io} - h_{id}|^{-\beta},$$
$$y_i \triangleq r_i^{-1/1.85}, \quad \beta \triangleq 0.85/1.85 \qquad (2.2)$$

where h_{io} and h_{id} denote the heads at the initial node and the terminal node of link i, respectively.

We define the differential resistance $\bar{r}_i^\nu$ and the differential conductance $\bar{y}_i^\nu$ at $q_i = q_i^\nu$ by

$$\bar{r}_i^\nu \triangleq \partial h_i / \partial q_i |_{q_i = q_i^\nu} = 1.85 r_i |q_i^\nu|^{0.85} \tag{2.3}$$

and by its reciprocal, respectively.

It is noted that a pipe corresponds to a nonlinear resistor in EN.

2.4.2 Valves

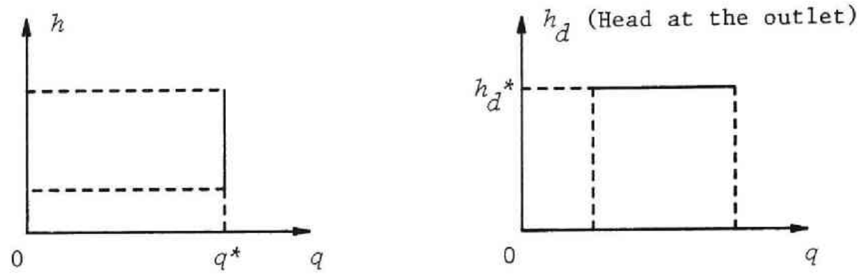
Various types of valves are in practical use for various purposes. But the fundamental characteristic of any valve can be represented by

$$h_i = r_{vi} q_i |q_i| \tag{2.4}$$

where r_{vi} is the resistance factor determined by an opening and a diameter of the valve.

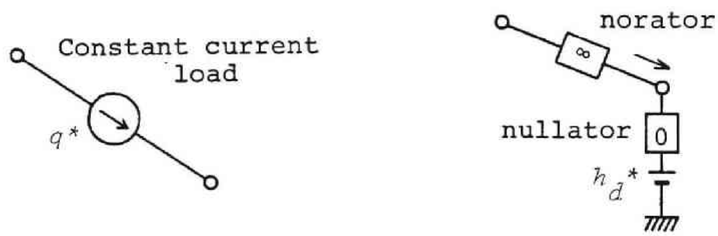
If the opening is fixed, the valve is equivalent to a nonlinear resistor in EN like the pipe, and the differential resistance and the differential conductance can be defined just like those of the pipe.

As a kind of special valves, a self-tuning valve changes its opening automatically, by some mechanical device or with aid of a local controller, so as to keep the flow rate or the head at its outlet unchanged. A self-tuning valve for keeping constant flow rate is called a flow regulating valve, and that for keeping constant head a head regulating valve. Characteristic curves of these valves are illustrated in Fig. 2.2. Hence the self-tuning valves correspond to the electric devices shown in Fig. 2.3.



(a) Flow regulating valve (b) Head regulating valve

Fig. 2.2. Characteristic curves of self-tuning valves.



(a) Equivalent to a flow regulating valve (b) Equivalent to a head regulating valve

Fig. 2.3. Equivalent electric devices to self-tuning valves.

A nullator and a norator are ideal devices for denoting singularities of EN: the former is a device whose voltage and current are both zero, and the latter is a device whose voltage and current are both arbitrary.

2.4.3 Pumps

Typical characteristics of fixed speed pumps for on-off control are shown in Fig. 2.4. Recently, however, variable speed pumps are getting popular. They achieve flexible and efficient operating conditions. For example, with aid of a

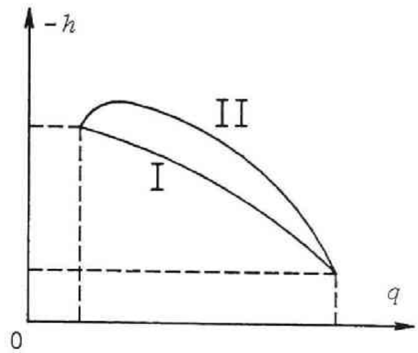
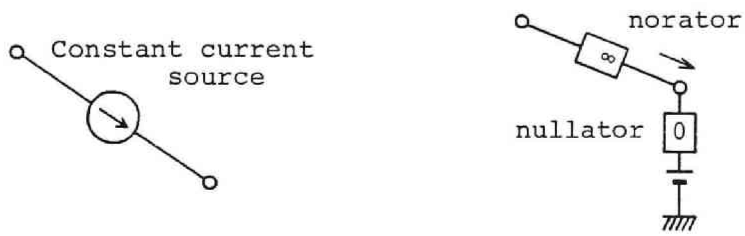


Fig. 2.4. Typical head-flow characteristics of pumps.



(a) Equivalent to a constant flow pump

(b) Equivalent to a constant head pump

Fig. 2.5. Equivalent electric devices to intelligent pumps.

local controller, the pump becomes an intelligent one: the local controller regulates speed of a driving motor so as to maintain the flow rate or the head at the outlet of the pump at a level specified by an operator or by a central computer. Equivalent electric devices to such intelligent pumps are shown in Fig. 2.5.

2.4.4 Reservoirs

Operation rate of a purification plant is ideally to be constant in order to reduce its construction cost. As consumption rate fluctuates considerably by time, installations for storing purified water are indispensable for keeping the operation rate constant. Such installations include service reservoirs, service towers, service tanks, and so on, which we call by a generic name of "reservoir".

In some problems of WDN, time variation of the water levels in reservoirs must be considered. In steady-state problems, however, the water levels are taken to be constant. Hence, the reservoir is equivalent to a constant voltage source in EN.

2.5 Flow Conservation Law and Head Difference Loop Law

The flow conservation law (FCL) and the head difference loop law (HDLL) are definite conditions to be satisfied at the steady state of WDN. The former corresponds to Kirchhoff's current law (KCL) and the latter to Kirchhoff's voltage law (KVL) in EN, respectively.

2.5.1 Flow conservation law (FCL)

FCL states that the incoming flow rate equals the outgoing flow rate at any node, or that the flow rate which crosses any cutset /2.3/ to the right equals the flow rate which crosses the same cutset to the left.

FCL can be described by use of a datum-node reduced incidence matrix or a fundamental cutset matrix of the network graph /2.3/. The fundamental cutset matrix is to be derived from a tree whose corresponding cotree includes all links with known flow rates.

Example-1 In the graph of Fig. 2.1(c), suppose that the flow rates of links III, IV, and V are known. Then FCL is written as follows:

$$
\begin{array}{cc}
 & \begin{array}{cc|ccccc|ccc} \mathrm{I} & \mathrm{II} & 1 & 2 & 3 & 4 & 5 & \mathrm{III} & \mathrm{IV} & \mathrm{V} \end{array} \\
\begin{array}{c} \mathrm{N} \\ \mathrm{O} \\ \mathrm{D} \\ \mathrm{E} \end{array}
\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \end{array} &
\left[\begin{array}{cc|ccccc|ccc}
1 & & 1 & & & & & & & \\
 & 1 & & 1 & & & & & & \\
\hline
 & & -1 & & 1 & 1 & & 1 & & \\
 & & & & -1 & & 1 & & 1 & \\
 & & & -1 & & -1 & -1 & & & 1
\end{array}\right]
\end{array}
\quad \text{Incidence matrix}
$$

$$
\times \left[\begin{array}{cc|ccccc|ccc} q_{\mathrm{I}} & q_{\mathrm{II}} & q_1 & q_2 & q_3 & q_4 & q_5 & q_{\mathrm{III}} & q_{\mathrm{IV}} & q_{\mathrm{V}} \end{array}\right]^T = 0
$$

or

$$
\begin{array}{cc}
 & \begin{array}{ccccc|ccccc} & & \text{Tree} & & & & & \text{Cotree} & & \\ \mathrm{I} & \mathrm{II} & 1 & 2 & 3 & 4 & 5 & \mathrm{III} & \mathrm{IV} & \mathrm{V} \end{array} \\
\begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ 1 \\ 2 \\ 3 \end{array} &
\left[\begin{array}{ccccc|ccccc}
1 & & & & & 1 & 1 & 1 & 1 & \\
 & 1 & & & & -1 & -1 & & & 1 \\
 & & 1 & & & -1 & -1 & -1 & -1 & \\
 & & & 1 & & 1 & 1 & & & -1 \\
 & & & & 1 & & -1 & & -1 &
\end{array}\right]
\end{array}
\quad \text{Fundamental cutset matrix}
$$

$$
\times \left[\begin{array}{ccccc|ccccc} q_{\mathrm{I}} & q_{\mathrm{II}} & q_1 & q_2 & q_3 & q_4 & q_5 & q_{\mathrm{III}} & q_{\mathrm{IV}} & q_{\mathrm{V}} \end{array}\right]^T = 0
$$

The superscript T denotes transposition of a vector or a matrix.

Such a tree as above can not be found if the flow rates of all the links joining at a node or constituting a cutset are known (measured or specified). In such a case, after checking that those flow rates satisfy FCL, assume the flow rate of a proper link to be unknown.

Example-2 In the graph of Fig. 2.1(c), suppose that all the flow rates of links I, II, III, IV and V are known and that their sum is zero. Then, the flow rate of any one of the five links is treated as an unknown.

$$
\begin{array}{cc}
 & \begin{array}{ccccc:ccccc} \multicolumn{5}{c}{\text{Tree}} & \multicolumn{5}{c}{\text{Cotree}} \\ \mathrm{I} & 1 & 2 & 3 & 4 & 5 & \mathrm{II} & \mathrm{III} & \mathrm{IV} & \mathrm{V} \end{array} \\
\begin{array}{c} \mathrm{I} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{ccccc:ccccc}
1 & & & & & & 1 & 1 & 1 & 1 \\
 & 1 & & & & & -1 & -1 & -1 & -1 \\
 & & 1 & & & & 1 & & & \\
 & & & 1 & & -1 & & & -1 & \\
 & & & & 1 & 1 & -1 & & & -1
\end{array}\right] \quad \begin{array}{l}\text{Fundamental} \\ \text{cutset} \\ \text{matrix}\end{array}
\end{array}
$$

$$
\times \left[\begin{array}{ccccc:ccccc} q_{\mathrm{I}} & q_1 & q_2 & q_3 & q_4 & q_5 & q_{\mathrm{II}} & q_{\mathrm{III}} & q_{\mathrm{IV}} & q_{\mathrm{V}} \end{array}\right]^T = 0
$$

2.5.2 Head difference loop law (HDLL)

HDLL states that the total head difference around any tieset (loop) is zero, and it can be described by use of a fundamental tieset matrix /2.3/. The tieset matrix is to be derived from a tree which includes all the links with known head differences.

Example-3 In the graph of Fig. 2.1(c), suppose that the head differences of links I and II are known. Then, HDLL is described as follows:

$$
\begin{array}{cc}
 & \begin{array}{ccccc:ccccc} \multicolumn{5}{c}{\text{Tree}} & \multicolumn{5}{c}{\text{Cotree}} \\ \mathrm{I} & \mathrm{II} & 1 & 2 & 3 & 4 & 5 & \mathrm{III} & \mathrm{IV} & \mathrm{V} \end{array} \\
\begin{array}{c} 4 \\ 5 \\ \mathrm{III} \\ \mathrm{IV} \\ \mathrm{V} \end{array} & \left[\begin{array}{ccccc:ccccc}
-1 & 1 & 1 & -1 & & 1 & & & & \\
-1 & 1 & 1 & -1 & 1 & & 1 & & & \\
-1 & & 1 & & & & & 1 & & \\
-1 & & 1 & & 1 & & & & 1 & \\
 & -1 & & 1 & & & & & & 1
\end{array}\right] \quad \begin{array}{l}\text{Fundamental} \\ \text{tieset} \\ \text{matrix}\end{array}
\end{array}
$$

$$
\times \left[\begin{array}{ccccc:ccccc} q_{\mathrm{I}} & q_{\mathrm{II}} & q_1 & q_2 & q_3 & q_4 & q_5 & q_{\mathrm{III}} & q_{\mathrm{IV}} & q_{\mathrm{V}} \end{array}\right]^T = 0
$$

If a set of links with known head differences constitutes a loop, the head difference of a proper link in the set must be dealt as an unknown quantity.

The full set of meshes can also be used as a set of loops for describing HDLL, which will be discussed in Chapter 3.

In Examples 1 and 3, the cutset matrix and the tieset matrix are derived from the same tree, as is usual in the past works on water networks analysis. In more general cases, however, it is obviously impossible to derive those two matrices from the same tree: if both the head difference and the flow rate of a link are known, the link must be included in a cotree for deriving a cutset matrix, while in a tree for deriving a tieset matrix. This is one of the basic concepts of the two-graph method /2.4/ which is originally developed for analysis of active electrical networks containing mutually coupled elements, especially control sources, which may or may not generate energy /2.5/. Use of the method enables us to refine formulations of a variety of problems in WDN, which are otherwise impossible.

In each of the succeeding chapters, FCL and HDLL are represented by matrix-vector notations suitably adapted to the problem of the chapter.

CHAPTER 3 A COMPREHENSIVE STUDY ON THE STEADY-STATE FLOW ANALYSIS OF WATER DISTRIBUTION NETWORKS

3.1 Introduction

So far, the steady-state flow analysis (hereafter abbreviated as SSFA) has been done to obtain the inflow rates from sources (reservoirs) and the water heads at demand terminals, prescribed with the heads at sources, the outflow rates (consumptions) at demand terminals, and all link characteristics including pipes, pumps and valves. We call such SSFA the basic SSFA. The basic SSFA is reduced to formulating and solving a set of nonlinear simultaneous algebraic equations, and a variety of methods for formulation and solution have been proposed so far /3.1/-/3.12/.

However, systematic discussions have been lacking for examining those methods of formulation, methods of fast solution, and further desirable linked usages among them. Water distribution networks are getting larger nowadays as a result of growth of city areas and/or integration of networks. Since a number of SSFA are being made in optimization processes of design and/or control, it is an urgent need to establish the most efficient method for fast solution of large-scale problems.

Moreover, in several of real applications to design and/or control problems, more freedom is required in choosing quantities to be prescribed and to be calculated. For example, there exist problems in which some of the characteristic values of conduit elements such as the heads of pumps, the openings of valves and the diameters of pipes are unknowns, or the inflow rates from the existing reservoirs as well as their water levels are measured and then known quantities. Mathematical treatment of self-tuning valves and intelligent pumps also require a modification of SSFA. A trial of such modification was made by Shamir /3.1/. However, his formulation is effective only for introducing resistance factors of pipes as unknowns, and his discussion on the solvability condition lacks mathematical strictness.

The first part of this chapter discusses fundamentals of the generalized SSFA which is favorable for study of a wide class of real design and/or control problems. The second part develops a systematic investigation on the basic SSFA: (1) redefining the basic SSFA, (2) presenting systematic procedures of formulation, (3) characterizing the existing formulations, (4) examining efficiency of typical solution methods, i.e., the Newton-Raphson method, the Hardy Cross method and their modifications, in connection with the presented formulations, and (5) utilizing the sparse matrix technique for fast solution. The third part is concerned with the generalized SSFA: a practically efficient linkage of methods for formulation and solution is presented, and then some illustrative examples are discussed.

3.2 Fundamentals of the Generalized SSFA

3.2.1 Classification of links

In formulating the generalized SSFA and in diagnosing its solvability, it is necessary to classify various links of a network into five sets shown in Table 3.1.

Table 3.1. Classification of links

Category	Head differential	Flow Rate	Characteristic	Number of links
E_b	○	○	◌	n_b
E_e	○	×	×	n_e
E_j	×	○	×	n_j
E_u	×	×	×	n_u
E_k	×	×	○	n_k

○ : known, × : unknown

It should be noted that the links are classified not by their physical properties but by whether or not their head differences, flow rates and/or characteristics (element-values) are known. The word "known" can be replaced by the word "specified" as well as by the word "measured".

Some examples of the classification are shown in the following.

A reservoir link, if only its head difference is known, is classified into E_e, if only its flow rate is known, into E_j, if both of them are known, into E_b, if neither of them is known, into E_u. The same can be said of a consumption link.

A pipe link, if only its characteristics are known, is classified into E_k, if its head difference and/or flow rate in addition to its characteristics are known, into E_b. The same can be said of a link of a fixed opening valve and that of a fixed speed pump.

A link of a flow regulating valve whose flow rate is known is put into E_j.

As to a link of an intelligent pump whose outlet head is specified, the link itself is classified into E_u, and the in/outflow link of its outlet side is changed from E_u to E_e or from E_j to E_b. The head specified to the pump is used for the known head of the in/outflow link.

A link whose characteristic is to be determined, e.g., a flow regulating valve whose flow rate is to be determined, or a pipe whose diameter is to be determined, is classified into E_u. Since the head difference h_i and the flow rate q_i of link i in E_u are calculated through the analysis, the flow rate of the flow regulating valve is obtained directly as q_i, while the diameter of the pipe is obtained by solving Eq. (2.1) with respect to r_i and d_i.

Now, let us denote the characteristics of E_k-links by a general form of

$$h_i = f_i(q_i) \tag{3.1}$$

or

$$q_i = g_i(h_i) \tag{3.2}$$

$$q_i = g_i(h_{io}-h_{id}) \tag*{(3.2)'}$$

Then a differential resistance $\bar{r}_i^{\nu}$ and a differential conductance $\bar{y}_i^{\nu}$ at $q_i=q_i^{\nu}$ are given by

$$\bar{r}_i^{\nu} = \partial h_i / \partial q_i |_{q_i = q_i^{\nu}} = \partial f_i(q_i) / \partial q_i |_{q_i = q_i^{\nu}} \tag{3.3}$$

and by its reciprocal, respectively.

As for conduit-element link i in E_b or in E_e, the relation

$$h_{io} - h_{id} = h_i \tag{3.4}$$

holds, where h_i is the known value.

3.2.2 Flow conservation law and head difference loop law

First, the flow conservation law (FCL) can be written in matrix notation as

$$AQ = 0 \qquad A\text{: datum-node reduced incidence matrix} \tag{3.5}$$

or

$$\underbrace{\begin{bmatrix} I_e & & & \Gamma_{e,kc} & \Gamma_{e,b} & \Gamma_{e,j} \\ & I_u & & \Gamma_{u,kc} & \Gamma_{u,b} & \Gamma_{u,j} \\ & & I_{kt} & \Gamma_{kt,kc} & \Gamma_{kt,b} & \Gamma_{kt,j} \end{bmatrix}}_{\text{Fundamental cutset matrix}} \begin{bmatrix} Q_e \\ Q_u \\ Q_{kt} \\ \hline Q_{kc} \\ Q_b \\ Q_j \end{bmatrix} \begin{matrix} \text{Tree} \\ \\ = 0 \text{ - - -} \\ \\ \text{Cotree} \end{matrix} \tag{3.6}$$

A fundamental cutset matrix is derived from a tree which includes all E_e and E_u links and whose corresponding cotree includes all E_b and E_j links. $Q_{\cdot}$ denotes the vector of flow rates of all links in each set, and $I_{\cdot}$ an identity matrix. The subscripts kt and kc denote E_k links included in the tree and the cotree, respectively.

Second, the head difference loop law (HDLL) can be written in matrix notation as

$$\underbrace{\begin{bmatrix} B_{kc,b} & B_{kc,e} & B_{kc,kt} & I_{kc} & & \\ B_{u,b} & B_{u,e} & B_{u,kt} & & I_u & \\ B_{j,b} & B_{j,e} & B_{j,kt} & & & I_j \end{bmatrix}}_{\text{Fundamental tieset matrix}} \begin{array}{c} \begin{bmatrix} H_b \\ H_e \\ H_{kt} \\ \hline H_{kc} \\ H_u \\ H_j \end{bmatrix} \end{array} \begin{array}{l} \text{Tree} \\ \\ = 0 \; --- \\ \\ \text{Cotree} \end{array} \qquad (3.7)$$

A fundamental tieset matrix is derived from a tree which includes all E_b and E_e links and whose corresponding cotree includes all E_u and E_j links. $H.$ denotes the vector of head differences of all links in each set.

It should be noted that the partitioning of E_k links into E_{kt} and E_{kc} in Eq. (3.6) is identical with that in Eq. (3.7). If such a pair of trees can not be found, the problem is not solvable. Namely, the following topological condition, which is equivalent to that for the solvability of an active electrical network, is not satisfied.

<u>Topological condition for the solvability</u> /3.13/-/3.14/

Obtain a flow graph G_{qk} and a head graph G_{hk} from G as follows.

G_{qk}: Deleting E_b and E_j links and then contracting E_e and E_u links.

G_{hk}: Deleting E_u and E_j links and then contracting E_b and E_e links.

The topological condition for the solvability of steady-state equations is the existence of a common tree of G_{qk} and

G_{hk}. The common tree is a set of links which composes a tree of G_{qk} as well as that of G_{hk}. In addition, it can be easily derived that

$$n_b = n_u \tag{3.8}$$

is a necessary condition.

An algorithm for searching a common tree also can be found in /3.13/.

3.3 Definition, Formulation and Solution of the Basic SSFA

On the basis of aforementioned fundamentals of the generalized SSFA, we develop a systematic study of the basic SSFA in this section.

3.3.1 Definition of the basic SSFA

As is discussed in the preceding section, in the generalized SSFA, the fundamental cutset matrix for FCL and the fundamental tieset matrix for HDLL are derived from two distinct trees in general. To the contrary, only one tree has been used for the derivation of those matrices in the past works on SSFA, then the analysis is feasible when both E_b and E_u are empty sets. Hence, the basic SSFA can be defined as a method applicable to problems where neither E_b links nor E_u links exist. In conformity with this definition, the basic SSFA may deal with the conduit-element link in E_e or E_j, the reservoir link in E_j and/or the consumption link in E_e.

The conventional methods for SSFA, however, have been entirely concerned with a more restricted case in which the whole set of reservoir links is identical with E_e, that of consumption links with E_j, and that of conduit-element links with E_k. Hence, in the following, systematic procedures of formulation are presented for this case. The discussion can be readily extended to cope with all problems in the scope of the above definition.

By the way, the basic SSFA is equivalent to the analysis of an electrical network which is composed of nonlinear resistors, voltage sources and/or current sources. Therefore, if all the pump characteristics are strictly monotone functions represented like curve I in Fig. 2.3, the solution is unique /3.15/. It is noted that, even if a pump curve is like curve II, a synthesized curve with that of its series pipe generally becomes like curve I.

3.3.2 Flow conservation law and head difference loop law

In the basic SSFA, FCL and HDLL are simplified as follows:

(a) Flow conservation law

$$\begin{array}{c} n_e \\ n_j \end{array} \begin{bmatrix} I_e & A_{e,kt} & A_{e,kc} & 0 \\ 0 & A_{j,kt} & A_{j,kc} & I_j \end{bmatrix} \begin{bmatrix} Q_e \\ Q_{kt} \\ \hline Q_{kc} \\ Q_j \end{bmatrix} \begin{array}{l} \text{Tree} \\ = 0 \\ \text{Cotree} \end{array} \tag{3.9}$$

$$\begin{array}{cccc} n_e & n_j & m-n_j & n_j \end{array}$$

Incidence matrix

or

$$
\begin{array}{c}
n_e \\ n_j
\end{array}
\begin{bmatrix} I_e & 0 & \Gamma_{e,kc} & \Gamma_{e,j} \\ 0 & I_{kt} & \Gamma_{kt,kc} & \Gamma_{kt,j} \end{bmatrix}
\begin{bmatrix} Q_e \\ Q_{kt} \\ \hdashline Q_{kc} \\ Q_j \end{bmatrix}
\begin{array}{l} \text{Tree} \\ = 0 \text{ ----} \\ \text{Cotree} \end{array} \qquad (3.10)
$$

$$
n_e \quad n_j \quad m-n_j \quad n_j
$$

Fundamental cutset matrix

where, the size of each submatrix is shown on left and below of the coefficient matrix. It is not necessary to decompose E_k into E_{kt} and E_{kc} in Eq. (3.9); the decomposition is for clarifying the relationship with other equations.

(b) Head difference loop law

$$
\begin{array}{c}
m-n_j \\ n_j
\end{array}
\begin{bmatrix} B_{kc,e} & B_{kc,kt} & I_{kc} & 0 \\ B_{j,e} & B_{j,kt} & 0 & I_j \end{bmatrix}
\begin{bmatrix} H_e \\ H_{kt} \\ \hdashline H_{kc} \\ H_j \end{bmatrix}
\begin{array}{l} \text{Tree} \\ = 0 \text{ ----} \\ \text{Cotree} \end{array} \qquad (3.11)
$$

$$
n_e \quad n_j \quad m-n_j \quad n_j
$$

Fundamental tieset matrix

E_{kt} and E_{kc} are the same sets as those in Eqs. (3.9) and (3.10)

It is evident from fundamental theorems of graph theory that the following relations hold:

$$
\begin{aligned}
\Gamma_{e,kc} &= -B^T_{kc,e} = A_{e,kc} - A_{e,kt}A^{-1}_{j,kt}A_{j,kc} \\
\Gamma_{e,j} &= -B^T_{j,e} = -A_{e,kt}A^{-1}_{j,kt} \\
\Gamma_{kt,kc} &= -B^T_{kc,kt} = A^{-1}_{j,kt}A_{j,kc} \\
\Gamma_{kt,j} &= -B^T_{j,kt} = A^{-1}_{j,kt}
\end{aligned} \qquad (3.12)
$$

The superscript T denotes transposition of a vector or a matrix.

In the following, another simple expression equivalent to the first row of Eq. (3.11) is given. First, a "mesh" of the network is defined /3.16/. An infinite plane is divided into an infinite region and some finite regions by a looped planar graph on the plane. A loop on the boundary of each finite region is called a mesh, and the whole set of the meshes is a set of necessary and sufficient loops of the graph.

In this dissertation, however, the word "meshes" is used for "the set of necessary and sufficient loops chosen so that the number of their composing links is made as small as possible". Then, the graph is not restricted to a planar one.

Let the graph obtained by deleting E_j links from G be G^*, and let the mesh matrix of G^* be $M=(m_{ij})$. M is a matrix where each row corresponds to each mesh and each column to each link, and $m_{ij}=1$ if link j is in mesh i in the same orientation as the mesh, $m_{ij}=-1$ if link j is in mesh i in the opposite orientation to the mesh, $m_{ij}=0$ if link j is not in mesh i.

Then, an alternative expression of HDLL equivalent to the first row of Eq. (3.11) can be given as:

$$M \left[H_e^T \; H_{kt}^T \; H_{kc}^T \right]^T = 0 \tag{3.13}$$

3.3.3 Formulation

The basic SSFA is to deal with a set of nonlinear simultaneous equations derived from FCL, Eq. (3.9) or (3.10), HDLL, Eq. (3.11) or (3.13), and the characteristics of E_k links, Eq. (3.1) or (3.2) or (3.2)'. The size of the simultaneous equations can considerably be reduced by eliminating proper unknowns, as is discussed in the following.

(a) Node-head method

Substitution of Eq. (3.2)' into the second row of Eq. (3.9) yields $n_j(=m-n_e)$-simultaneous equations whose unknowns are all the components of H_j (the heads at nodes).

(b) Cotree-flow method

Substitution of Eq. (3.1) into the first row of Eq. (3.11) and elimination of Q_{kt} thereupon by using the second row of Eq. (3.10) leads to $(m-n_j)$-simultaneous equations whose unknowns are all the components of Q_{kc} (the flow rates of cotree links).

(c) Tree head difference method

Substitution of Eq. (3.2) into the second row of Eq. (3.10) and elimination of H_{kc} thereupon by using the first row of Eq. (3.11) gives n_j-simultaneous equations whose unknowns are all the components of H_{kt} (the head differences of tree links). In spite of an additional effort required for the derivation of a cutset matrix, the number of unknowns is equal to that of the node-head method. Hence, we exclude this formulation in the following discussion.

(d) Mesh-flow method

A mesh-flow is defined as the constituent flow rate common to all the links in a mesh. Let us denote the mesh-flow rates by Q_m. Then the flow rates of E_e, E_{kt} and E_{kc} links are given by

$$[\ Q_e^T\ Q_{kt}^T\ Q_{kc}^T\ ]^T = M^T Q_m - [\ (\Gamma_{e,j} Q_j)^T\ (\Gamma_{kt,j} Q_j)^T\ 0\ ]^T \quad (3.14)$$

By substituting Eq. (3.1) into Eq. (3.13) and using Eq. (3.14), we obtain $(m-n_j)$-simultaneous equations whose unknowns are Q_m (the mesh-flow rates).

The second term on the right side of Eq. (3.14) denotes the flow pattern obtained by supposing that all consumptions are conveyed from the head-known nodes through only the tree links. Let us call the flow rate in each link of the pattern the "fixed-flow rate".

3.3.4 Solution

The Newton-Raphson method (hereafter abbreviated as NR method) and the Hardy Cross method (HC method) are the most well-known and widely used iteration methods for solving nonlinear simultaneous equations.

(a) Newton-Raphson method

Given a set of n-simultaneous equations as

$$F(X) \triangleq (f_1(X),\ f_2(X), \ldots,\ f_n(X)) = 0,$$

$$X \triangleq (x_1,\ x_2, \ldots,\ x_n) \tag{3.15}$$

NR method, starting from a properly given initial value X^0 of X, iterates corrections due to equations

$$J^\nu \Delta X^\nu = -F(X^\nu) \qquad \Delta X^\nu\text{: correction vector}$$

$$\text{(Correction equations)} \tag{3.16}$$

and

$$X^{\nu+1} = X^\nu + \Delta X^\nu \tag{3.17}$$

until $F(X^{\nu+1}) \cong 0$ is satisfied. In the above, J^ν denotes the value of the Jacobian matrix (J-matrix) $J=(j_{ij})=\partial F/\partial X=(\partial f_i/\partial x_j)$ evaluated at the current point $X=X^\nu$.

J^ν-matrix of each formulation of the basic SSFA is given by

(Node-head method) $\quad [\, A_{j,kt}\ A_{j,kc} \,]\ \overline{Y}^\nu_k\ [\, A_{j,kt}\ A_{j,kc} \,]^T$ (3.18)

(Cotree-flow method) $\quad B_{kc,kt}\overline{R}^\nu_{kt}B^T_{kc,kt}+\overline{R}^\nu_{kc}$ (3.19)

(Mesh-flow method) $\quad M^*\overline{R}^\nu_k(M^*)^T$ (3.20)

where

$$\overline{Y}^\nu_{\cdot} \triangleq \underset{i\in E.}{\mathrm{diag}}(\overline{y}^\nu_i), \qquad \overline{R}^\nu_{\cdot} \triangleq \underset{i\in E.}{\mathrm{diag}}(\overline{r}^\nu_i)$$

The superscript ν implies that the values are evaluated at $h_i=h^\nu_i$ $(q_i=q^\nu_i)$, and $E.$ denotes a proper set. M^* is a matrix obtained by removing the columns of E_e links from M.

Table 3.2. Principal features of the formulations of the basic SSFA

Formulation		Node-head method	Cotree-flow method	Mesh-flow method
Size of the equation		$n_j = n - n_e$	$m - n_j$	
J-matrix	Sparsity*, Diagonal dominance	Dominant in general [1)]	Depend on the tree spanned [2)]	Dominant in general [3)]
	Positive definiteness, Symmetry	Hold		
Initial values of the unknowns		Heads at the both ends of any link should not be the same	As for any tieset, the flow rate of at least one link should be nonzero	

*) The number of nonzero elements in each row of J-matrix is

1) The number of the links joining at each node plus 1
2) The number of the tiesets that commonly own more than one link with each tieset plus 1
3) The number of the meshes adjacent to each mesh plus 1

The important features of the presented formulations are summarized as in Table 3.2. The conditions on initial values of the unknowns are necessary for making the elements of J-matrix finite and the matrix nonsingular.

(b) Hardy Cross method

HC method was originally developed for simple hand calculation. It simplifies the correction equations, Eq. (3.16), by putting all off-diagonal elements of J^{ν} to zero. Namely, the correction vector ΔX^{ν} is calculated by

$$\Delta X^{\nu} \triangleq (\Delta x_i^{\nu}) = (-f_i(X^{\nu})/j_{ii}^{\nu}) \tag{3.21}$$

Hence, the method reduces the solution procedure of linear simultaneous equations to simple divisions, and then saves the storage when implemented in a computer. However, as a matter of course, the number of iterations increases, and divergence or oscillation of the solution is apt to be caused.

3.3.5 Characterization and evaluation of the existing methods

Now, we try to pigeonhole typical existing methods for the basic SSFA referring to the presented formulations and solutions. First, we consider those introduced in Ref. /3.2/.

Both the Marlow method and the Martin-Peters method consist of the formulation by the node-head method and the solution by NR method. In the Marlow method, however, the correction vector ΔX^{ν} is 1/1.85 times that given by Eq. (3.16).

The loop-equation method by Takakuwa consists of the cotree-flow method and NR method.

Formulation by Yasuno and that by Goda-Okura are elementary versions of our flow methods, and because of no use of the relation Eq. (3.10), they are redundant: the flow rates of all the links in E_e and E_k are unknowns. The difference between the two methods consists in whether Eq. (3.16) is written with respect to ΔX^{ν} or to $X^{\nu+1}$.

The Cross method consists of the mesh-flow method and HC method.

Further, the method by Epp et al. /3.5/ consists of the mesh-flow method and NR method. It saves computer storage by making J-matrix a banded one and improves the speed of solution by producing good initial values of the unknowns.

In the method by Wood et al. /3.7/ whose title has the adjective phrase of "using linear theory", the equations consist of Eqs. (3.9) and (3.13), while the unknowns are the flow rates of all the links in E_e and E_k. The solution is no more than NR method except that ΔX^{ν} is 1.85 times that given by Eq. (3.16).

3.4 Methods for Fast Solution of the Basic SSFA

For fast solution of the basic SSFA, it is necessary to reduce both the number of iterations until convergence and the time needed for one iteration. This section is concerned with these subjects.

3.4.1 Methods for improving the rate of convergence

NR method and HC method do not necessarily show satisfactory rates of convergence for the problems of the basic SSFA. Hence, a variety of their modifications have been proposed for improving the rate of convergence. In each of those modified methods, a multiplier α_i^{ν} is introduced into Eq. (3.17):

$$x_i^{\nu+1} = x_i^{\nu} + \alpha_i^{\nu}\Delta x_i^{\nu} \tag{3.22}$$

The following α_i^{ν}'s are examined in this section.

(A) $\alpha_i^{\nu} = 1 \quad (\forall\nu, \forall i)$: the basic method

(B) $\alpha_i^{\nu} = \begin{cases} 1 & (\Delta x_i^{\nu-1}\cdot\Delta x_i^{\nu} \geqq 0) \\ 0.5 & (\nu=0 \text{ or } \Delta x_i^{\nu-1}\cdot\Delta x_i^{\nu} < 0) \end{cases}$ /3.3/,/3.10/

(C) $\alpha_i^{\nu} = \begin{cases} 1 & (\nu\text{:even}) \\ \exp(\min(1,\ \Delta x_i^{\nu}/\Delta x_i^{\nu-1})) & (\nu\text{: odd}) \end{cases}$ /3.1/

(D) $\alpha_i^{\nu} = \alpha$: constant value $(\neq 1,\ \forall\nu,\ \forall i)$ /3.7/,/3.8/

(E) $\alpha_i^{\nu} = \lambda^{\nu} > 0 \ (\forall i)$: The value λ^{ν} is to minimize approximately $\phi(X^{\nu}+\lambda\Delta X^{\nu})$, where $\phi = F^T F$, and practically determined by the following simple line search: First, ϕ is evaluated for three distinct values of λ, and then ϕ is approximated to a quadratic function of λ by curve fitting. Then the value λ^{ν} is determined so as to minimize the quadratic function.

(F) The same as E-method except that the golden section search is used for determining λ^{ν}.

B-method suppresses the behavior of oscillating variables by halving their corrections. C-method, not only suppresses the oscillations, but also accelerates correction of the variables being successively corrected in the same direction. E-method and F-method are examined to clarify the relationship between accuracy of the line search and number of iterations in NR method.

Hereafter, each modified method will be identified by such a naming like "NR-B method" or "HC-C method", etc. Since E-method and F-method are much time consuming compared with the other methods, and the correction vector ΔX^{ν} of HC method is not so exact, HC-E method and HC-F method will not be examined.

In some past works, the effectiveness of each modified method was demonstrated in an empirical way. In the following, the efficiency of the modified methods are examined systematically in connection with the formulations presented in the previous section.

3.4.2 Features of the solution methods

A variety of networks have been analysed in order to examine the efficiency of the presented methods of solution in connection with the presented methods of formulation. Typical results are shown in Tables 3.3 through 3.6. The computation time in those tables indicates the time used only for the iterative process of NR method and HC method. Four networks of N1 to N4 are common in all the tables and their sizes are shown in Table 3.3. All the calculations are made on FACOM-M200 in the Data Processing Center of Kyoto University. Programs are coded in PL/I language and eight bytes are used for each real number.

The iteration is stopped when the error in every equation of FCL becomes less than $10^{-4}m^3/s$ for the node-head formulation, and when the error in every equation of HDLL becomes less than $10^{-4}m$ for the cotree-flow formulation and the mesh-flow formulation. In either case, the final values of the heads at nodes are correct to the third decimal place. In NR method, since the accuracy of the variables is improved by more than one digit per iteration in the neighborhood of the exact solution, a small difference in the reference value for stopping the iteration does not produce a significant difference in the number of iterations until convergence.

Initial values of the unknowns are set as follows: For the node-head formulation, the head at each node is set to the node height plus 20m. If the heads at two nodes connected by a link take the same value, either one of them is slightly changed. For the cotree-flow formulation, the cotree-flow rates are set to about 1.0% of the maximum value of the consumptions. For the mesh-flow formulation, the mesh-flow rates are chosen so that the cotree-flow rates take the same values as in the cotree-flow formulation.

The numerical value in parenthesis of D-method indicates

Table 3.3. Number of iteration and computation time (in parenthesis, millisecond) of NR method for the node-head formulation

Network		N1		N2	N3	N4
Number of nodes (n)		20	20	31	36	57
(Head-known nodes)		10	1	2	4	3
Number of links (m)		35	35	40	42	69
Number of unknowns		10	19	29	32	54
Methods	A	7(48)	8(71)	32(469)	37(645)	36(1914)
for	B	7(49)	8(72)	10(149)	14(242)	11(590)
improving	C	6(44)	9(82)	13(197)	10(180)	16(862)
convergence	D(0.5)	21(130)	21(175)	20(295)	19(336)	20(1070)
	E(0.5)	6(78)	6(94)	7(156)	7(175)	9(583)
	E(1.0)	6(78)	6(96)	6(134)	8(203)	8(524)
	F(0.1)	6(147)	6(170)	6(222)	7(268)	7(608)
	F(0.01)	6(186)	6(213)	6(275)	7(330)	8(829)

Table 3.4. NR method for the cotree-flow formulation

Network		N1	N2	N3	N4
Number of unknowns		16	11	10	15
Methods	A	8(143)	7(83)	6(64)	8(222)
for	B	8(159)	8(90)	7(73)	10(279)
improving	C	10(175)	8(90)	8(84)	13(349)
convergence	D(0.5)	22(371)	27(285)	22(217)	27(696)
	E(0.5)	8(223)	7(151)	5(104)	7(325)
	E(1.0)	7(197)	6(131)	5(104)	6(284)
	F(0.1)	7(363)	6(276)	5(230)	6(549)
	F(0.01)	7(451)	6(358)	5(300)	6(711)

Table 3.5. HC method for the cotree-flow formulation

Network		N1	N2	N3	N4
Number of Variables		16	11	10	15
Methods	A	OVF	*	OVF	OVF
for	B	63(106)	64(113)	659(1229)	208(632)
improving	C	*	200(371)	*	*
convergence	D(0.4)	130(251)	145(251)	963(1770)	487(1454)
	D(0.6)	85(140)	95(166)	*	320(945)

OVF: Overflow *: More than 1,000 (may be oscillating)

This table shows the case a tree is chosen so that the links of large resistance factor are included in the cotree, and that each link is contained in small number of tiesets as far as possible. In other cases, the solution rarely converges.

Table 3.6. HC method for the mesh-flow formulation

Network		N1	N2	N3	N4
Number of variables		16	11	10	15
Methods	A	77(128)	*	OVF	736(2226)
for	B	78(131)	72(131)	508(922)	735(2245)
improving	C	65(116)	71(130)	*	515(1614)
convergence	D(0.4)	194(323)	178(314)	798(1422)	*
	D(0.6)	129(216)	117(209)	*	*

OVF: Overflow *: More than 1,000 (may be oscillating)

Table 3.7. Typical sequences of λ in the line search

		Iteration					
Formulation	Network	1	2	3	4	5	6
Node-head method	N1	0.52	1.05	2.32	1.05	1.10	
	N2	0.62	0.58	1.14	1.01	1.00	
Cotree-flow method	N1	1.60	0.92	0.82	1.06	1.13	0.34
	N2	1.52	1.09	0.96	1.11	1.48	0.99

the value of α, that of E-method the middle value of three λ's at regular intervals of 0.5 for the quadratic curve fitting, whereas that of F-method the accuracy of λ in the golden section search.

(a) Newton-Raphson method

NR method converges in all cases. The correction equations are solved by the modified Cholesky method. In the following, only the results of the node-head formulation (Table 3.3) and the cotree-flow formulation (Table 3.4) are discussed. NR method presents the same rate of convergence for the cotree-flow and the mesh-flow formulations, when the initial values are equivalent. However, since sparsity of J-matrix of the mesh-flow formulation is higher than that of the cotree-flow formulation, the solution time of correction equations can be reduced by use of a sparse matrix technique in the mesh-flow formulation.

Now, let us compare the numbers of iterations. Apart from E-method and F-method, B-method and C-method give small values for the node-head formulation, while A-method for the cotree-flow formulation. E-method and F-method seem favorable for reducing the number of iterations. However, since they require a number of exponential computations for the line search, they do not necessarily shorten the solution time. Besides, as is evident from the comparison of E-method, F(0.1) method and F(0.01) method, the necessary time for the line search increases as its accuracy increases, and increase of the accuracy does not always lead to reduction in the number of iterations. Hence, for the line search, such a simple technique as E-method is

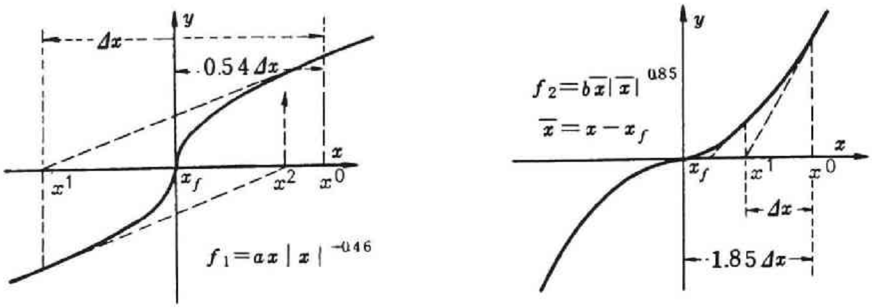


Fig. 3.1. Convergence of NR method.

recommended so far as it provides reasonable accuracy.

The reason why A-method (the basic method) gives slow convergence for the node-head formulation, while fast convergence for the cotree-flow formulation, is explained here. Let us consider the following equations with respect to a single variable x:

$$f_1(x) = a\bar{x}|\bar{x}|^{-\beta} = 0, \quad f_2(x) = b\bar{x}|\bar{x}|^{0.85} = 0$$

$$\bar{x} \triangleq x - x_f, \quad \beta \triangleq 0.85/1.85, \quad x_f\text{: the solution}$$

Figure 3.1 illustrates the solution procedure of these equations by NR method starting from the initial point x^0. It is evident that the solution x_f can be obtained by one iteration, if the correction Δx is multiplied by 1/1.85 for f_1 and by 1.85 for f_2. Likewise, NR method tends to produce over-corrections for the node-head formulation where the functions are composed of 1/1.85-th power of the unknowns and under-corrections for the cotree-flow formulation where the functions are composed of 1.85-th power of the unknowns.

Table 3.7 shows typical sequences of λ given by the line search (F(0.01) method). λ seldom exceeds 1.0 in the node-head formulation: it is about 0.5 to 0.7 in the first several iterations and about 1.0 in the succeeding iterations. In the cotree-flow formulation in contrast, it is about 1.5 to 1.85 in the first or first two iterations and about 1.0 in the succeeding iterations. This fact verifies the analogy between the convergence property of NR method for the single-variable functions and that for the multi-variable functions.

Hence, the dependence of the convergence rate of NR-A method on the formulations can be explained schematically by Fig. 3.1: For the cotree-flow formulation, although A-method produces under-corrections in early stages of the iteration, it brings the unknowns steadily to the solution. To the contrary, for the node-head formulation, A-method produces an oscillation which is hardly damped. In connection with this, B-method and C-method show fast convergence for the node-head formulation, because they are capable of suppressing the oscillation.

Now, we compare the computation time for the NR iterations. Let us decompose the computation time into $T_{\cdot}$, the time required for solving the correction equations, and $\Delta T_{\cdot}$, that required for the calculation of the multipliers $\alpha_{i\cdot}^{\nu}$, where $\cdot$ denotes each method. It is noted that ΔT_A is equal to zero, and ΔT_B, ΔT_C and ΔT_D are nearly zero.

For the cotree-flow formulation (and equivalently for the mesh-flow formulation), the basic NR method shows a fairly good convergence property as stated above, and T_A becomes always minimum: Although E-method and F-method reduce the number of iterations, and then T_E and T_F are smaller than T_A, $(T_E+\Delta T_E)$ and $(T_F+\Delta T_F)$ exceed T_A.

For the node-head formulation, B-,C- and E-methods compete in the solution time. However, C-method occasionally falls into slow convergence (see the column of N4), in other words, its convergence property is not so stable. Comparing B-method and E-method, T_E is smaller than T_B owing to the line search, but $(T_E+\Delta T_E)$ is approximately equal to T_B. When some sparse matrix technique is utilized for solving the correction equations, T_B and T_E are reduced in the same ratio, while ΔT_E does not change, making T_B less than $(T_E+\Delta T_E)$. Therefore, B-method is recommended for the node-head formulation.

(b) Hardy Cross method

HC method premises that a change in $f_i(X)$ is (almost) solely caused by a change in x_i. Therefore, it is effective for the node-head formulation and the mesh-flow formulation where the dominance of the diagonal elements of J-matrix are prominent, and gives endurable convergence rates for these formulations. In contrast, the convergence is poor for the cotree-flow formulation where the dominance is not so sharp. For supporting these statements, we examine the results of application to the mesh-flow formulation, Table 3.5, and the cotree-flow formulation, Table 3.6.

The difference between the mesh-flow formulation and the cotree-flow formulation is caused by the way how a full set of independent tiesets are chosen. Voyles et al. /3.4/ proposed to choose a set based on magnitude of the coefficient of resistance r_i of links. However, they merely illustrated it on an example network composed of the links plainly classified into some groups by the magnitudes of r_i's, and presented no concrete and general algorithm.

Among the modified HC methods, HC-B method shows relatively good convergence for the mesh-flow formulation. In general, however, whether the HC iteration converges or not is quite contingent on setting of initial values. Even when it converges, the number of iterations, usually not small, depends sensitively on a reference value for judging convergence.

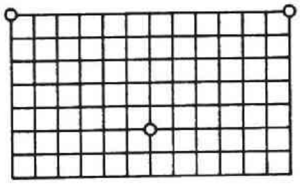
As to saving of the storage space, HC method can not be said quite superior to NR method, if some scheme for storing only nonzero elements in J-matrix is adopted in the latter.

In conclusion, HC method is generally not recommendable for use in the computer calculation.

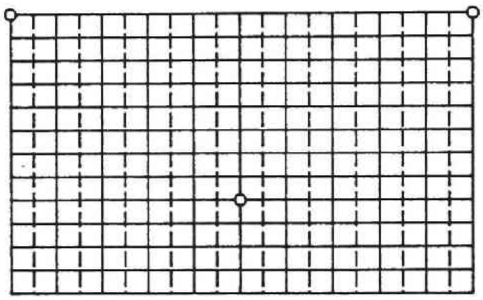
3.4.3 Examples of analysing large-scale networks

As we have seen, a linkage of the node-head formulation and NR-B method (hereafter simply called the Node-Head method), and a linkage of the mesh-flow formulation and NR-A method (the Mesh-Flow method) give fast solution of the basic SSFA. Since those formulations provide sparse Jacobian matrices, the computation time and the storage space necessary for solution of the correction equations can considerably be saved by use of the program of solving a system of linear equations with a symmetric sparse matrix (We have judged that NR-B method is superior to NR-E method for the node-head formulation, considering this advantage) /3.17/,/3.18/.

We have applied the Node-Head method and the Mesh-Flow method to the artificial large-scale networks shown in Fig. 3.2, for verifying the effectiveness of those methods. This subsec-

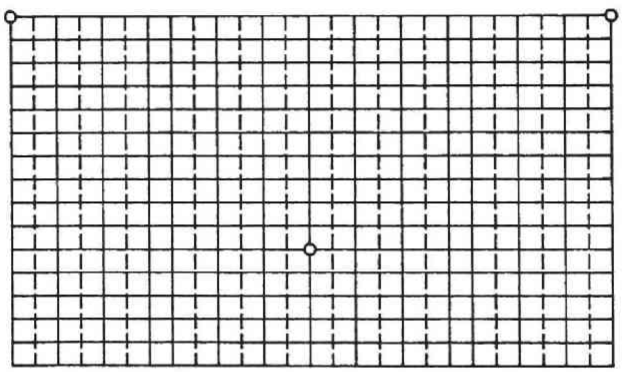


(a)



(b) With the links of broken lines

(c) Without the links of broken lines



(d) With the links of broken lines

(e) Without the links of broken lines

(f) Grid-like network with 20×33 nodes similar to (a), (b), (d)

Fig. 3.2. Large-scale networks (o: sources).

Table 3.8. Results of the application to large-scale networks

Network	(a)	(b)	(c)	(d)	(e)	(f)
Number of nodes	104	273	273	432	432	660
Number of links	187	512	392	821	641	1,267
number of meshes	86	242	122	392	212	610
Node-Head method (unit: ms)						
Iuput & output	88	238	202	394	332	598
Preparation	43	153	117	346	263	809
Solution time	130	827	794	2,586	2,490	5,787
(Number of iterations)	(8)	(10)	(10)	(14)	(14)	(14)
per iteration	16	83	79	185	178	413
Total time	261	1,218	1,113	3,316	3,085	7,194
Mesh-Flow method (unit: ms)						
Input & output	88	238	202	394	332	
Preparation	65	328	218	792	525	
Solution time	117	716	312	1,430	852	
(Number of iterations)	(8)	(9)	(8)	(8)	(9)	
per iteration	15	80	39	179	95	
Total time	270	1,282	732	2,616	1,709	

tion discusses the results of application shown in Table 3.8.

All the calculations are made by FACOM-M382 (MIPS value is 23) in the Data Processing Center of Kyoto University. Programs are coded in FORTRAN77 language and eight bytes are used for each real number.

Conditions for stopping the NR iteration are same as those used in the previous section.

Initial values of the unknowns are chosen as follows: For the Node-Head method, the head at each node is determined by the distance (depth) of the node from the nearest known-head node and its known-head; a smaller value is given as the distance becomes larger. The distance can be obtained by spanning a breadth-first tree whose root is the datum node in G^*. For the Mesh-Flow method, a breadth-first tree is also used to determine the fixed-flow rate in each link, and mesh-flow rates are provided with mutually different small values between $0.001 m^3/s$ and $0.003 m^3/s$. Then the closer a tree link is to its precedent known-head node, the greater value takes the flow-rate in the link. The above settings of the initial values are reasonable in the sense that those are fairly close to the pattern of the solution.

Now, we examine the computation time.

(a) Input and output (I/O)

The time for I/O is proportional to the sum of the numbers of nodes and links. Since the format of I/O is quite same in both the methods, it takes also the same value in both the methods.

(b) Data check and graph theoretical processing (preparation)

The times for check of duplication of the node name and the link name are proportional to the second power of the number of nodes and that of links, respectively. For the graph theoretical processing, the Node-Head method consumes quite few time because of its simplicity, while the Mesh-Flow method consumes a considerable time for searching the meshes.

In both methods, if the topology of the objective network is unchanged, the time for preparation can be saved, except the first execution, by storing the results of the first execution on an external storage.

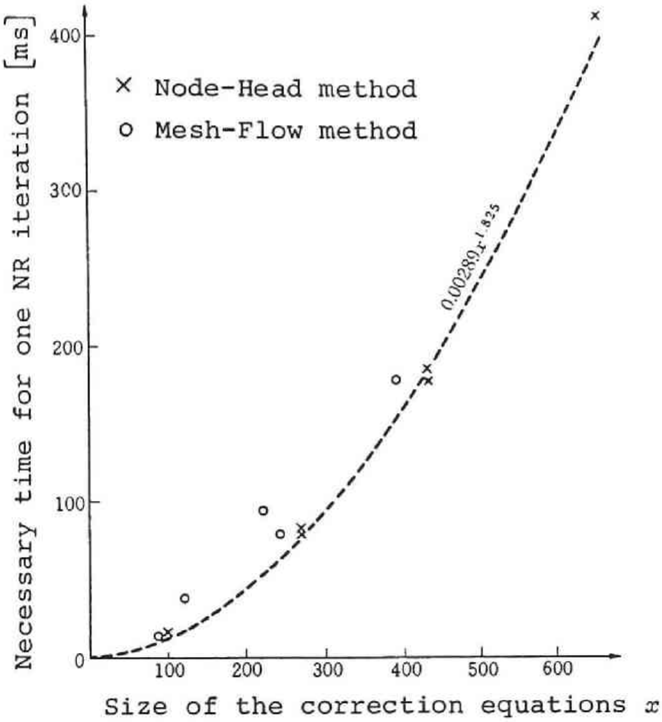


Fig. 3.3. Necessary time for one NR iteration.

(c) The computation time per iteration (Fig. 3.3)

The computation time per iteration is proportional to the 1.82-th power of the size of the correction equations. Hence it is clear that the computation time can considerably be saved by application of a sparse matrix technique, because the computation time of the Gauss elimination without a sparse matrix technique is proportional to the third power of the size of the equations in the worst case.

If the size of the equations is equal, the computation time per iteration of the Mesh-Flow method is a little larger than that of the Node-Head method. The reason is that the quantity of calculation about each mesh is generally a little larger than that about each node (Note that the number of links composing a mesh is generally larger than that of links joining at a node).

By the way, in the Mesh-Flow method, the number of links composing a mesh generally becomes large if the mesh includes source links. The variable and the equation of such a mesh must be put in a higher numbered column (and row) of the equations. Otherwise, the so called fill-in is frequently caused in solving the correction equations, and computation time increases.

(d) Number of iterations

The number of iterations in the Node-Head method is larger than that in the Mesh-Flow method. Although the results are not shown, for a poor initial setting, more than 15 iterations are required for the Node-Head method, while 12 iterations are enough for the Mesh-Flow method in the worst case.

(e) Total time for solution

The total solution time by the Node-Head method is slightly shorter than that by the Mesh-Flow method for (a) and (b) in Fig. 3.2, where only little difference is between the number of nodes and that of meshes. In contrast, the Mesh-Flow method solves (c), (d) and (e) fairly faster than the Node-Head method because of relatively small number of meshes in (c) and (e), and because of relatively small number of iterations in (d). In most real networks, the number of meshes are relatively smaller than that of nodes, then the Mesh-Flow method is more favorable for fast solution. If the topology of the objective network is unchanged, since the step of the preparation can be skipped except the first execution, the Mesh-Flow method has perfectly the precedence over the Node-Head method.

3.5 Formulation and Solution of the Generalized SSFA

In this section, we proceed to discussion of the generalized SSFA. The generalized SSFA is also composed of formulating and solving a set of nonlinear simultaneous algebraic equations. Hence, a variety of possible linkages of methods for formulation and solution have been comparatively studied again.

In the following, after three procedures of formulation are introduced, some notable results of a comparative study are summarized.

3.5.1 Formulation of the generalized SSFA

(a) Node-equation method

By substituting Eq. (3.2)', the characteristic equation of an E_k link, into Eq. (3.5), and thereupon using Eq. (3.4) for a conduit-element link in E_b and E_e, we obtain n-simultaneous nonlinear equations. The unknowns are $(n_e^c+n_e^n+n_u^c+n_u^n)$ flow rates of E_e and E_u links, and $(n_j^c+n_u^c-n_e^n-n_b^n)$ head differences of the in/outflow links. $n_\cdot^c$ and $n_\cdot^n$ denote the number of in/outflow links and that of conduit-element links in each set, respectively. The total number of the unknowns is equal to n as shown in the following:

$$
\begin{aligned}
& n_e^c + n_u^c + n_e^n + n_u^n + n_j^c + n_u^c - n_e^n - n_b^n \\
&= n_e^c + n_u^c + n_j^c + n_u^n + n_u^c - n_b^n \\
&= n_e^c + n_u^c + n_b^c + n_j^c \quad (n_u^c + n_u^n = n_b^c + n_b^n \text{ : Eq. (3.8)}) \\
&= n
\end{aligned}
$$

(b) Head difference method

By substituting Eq. (3.2), the characteristic equation of an E_k link, into the third row of Eq. (3.6), and thereupon by eliminating H_{kc} due to the first row of Eq. (3.7), we obtain $(n-n_e-n_u)$-simultaneous nonlinear equations whose unknowns are all the components of H_{kt}. Once the value of H_{kt} is found, the values of H_{kc}, H_u and H_j can readily be calculated through Eq. (3.7).

(c) Flow method

Substituting Eq. (3.1) into the first row of Eq. (3.7) and eliminating Q_{kt} due to the third row of Eq. (3.6), we obtain $(m-n_b-n_j)$-simultaneous nonlinear equations whose unknowns are all the components of Q_{kc}. Once the value of Q_{kc} is found, the values of Q_e, Q_u and Q_{kt} can readily be calculated through Eq. (3.6).

3.5.2 Recommended linkage of methods for formulation and solution

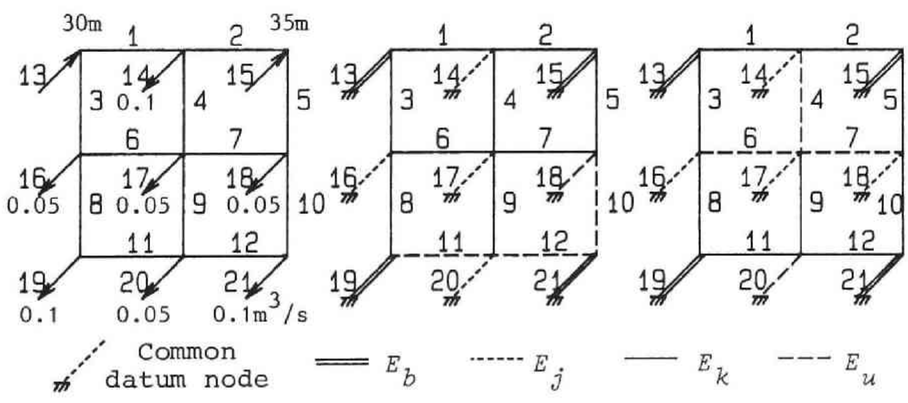
The efficiency of NR method and its modifications is examined in connection with the methods of formulation presented in the former subsection. And it is found that the linkage of the node-equation formulation and NR-B method is the most efficient one. The linkage is efficient because it is applicable to large-scale networks and it requires a relatively short time for solution: the node-equation formulation does not require the search of a common tree of G_{qk} and G_{hk} which is time-consuming for large-scale networks (Note that if the problem is not solvable, then the correction equations of NR method are also not solvable. Hence it is not necessary to check if a common tree exists or not); Further J-matrix is sparsely structured (Note that the number of nonzero elements in each row is equal to the number of the links joining at the corresponding node plus one). The convergence property of NR method for the node-equation formulation is similar to that for the node-head formulation of the basic SSFA.

It is noted that J-matrix is not symmetric for the generalized SSFA.

3.5.3 Examples

Some examples of the generalized SSFA are shown in this subsection.

(Ex1) Consider the network of Fig. 3.4(a). The heads at sources and the consumptions at demand nodes are given as shown in the figure. In addition, links 1 to 12 are supposed to be the pipes of 120 in smoothness coefficient, 200m in length and 0.25m in diameter. Then the inflow rate from source 13 is calculated to be $0.19 m^3/s$, and that from source 15, $0.31 m^3/s$. This example is nothing but that of the basic SSFA.



(a) Ex1, Ex2 (b) Ex3 (c) Ex4

Fig. 3.4. Examples of the generalized SSFA.

(Ex2) In the network of Fig. 3.4(a), consider to change the head at source 15 so that the inflow rate from source 13 becomes $0.2m^3/s$ and that from source 15 becomes $0.3m^3/s$, without changing other conditions. To this end, source 13 is supposed to be an E_b-link of 30m in head and $0.2m^3/s$ in flow rate, while source 15 an E_u-link. Then the head at source 15 is calculated to be 34.16m.

(Ex3) Consider to change the diameters of links 10, 11 and 12 from those in Ex1 in order to lower the head of consumption link 21 from 22.02m to 20m. The heads of links 13, 15 and 19, and the flow rates in all the in/outflow links are kept unchanged from those in Ex1. For this problem, the links are classified as shown in Fig. 3.4(b). Then, the diameters of links 10, 11 and 12 are calculated to be approximately 0.225m, 0.25m and 0.125m, respectively.

(Ex4) Let us consider the problem in which the knowns and the unknowns are given as shown in Fig. 3.4(c). For the knowns, the prescribed values or the obtained values in Ex1 are utilized. Calculations are started from a variety of initial points and four different solutions are obtained. In one of the solutions, links 4, 6 and 7 turn out to be the pipes of 0.25m in diameter, while in others, one or two of those links are found to be the pump. The uniqueness of the solution is not assured because the strict monotone is not supposed for E_u-links. Then a more investigation is needed to know how to attain the most practically favorable solution when the soluition is not unique.

3.6 Concluding Remarks

In this chapter, a comprehensive study on the SSFA has been developed with the aims of (1) generalizing the SSFA so that it allows more freedom in choosing quantities to be prescribed and to be calculated than the traditional (basic) SSFA does, and (2) establishing the most efficient method for fast solution of the SSFA.

In the first part of this chapter, the fundamentals of the generalized SSFA, i.e., the classification of links, the expressions of the flow conservation and the head difference loop laws, and the topological condition for the solvability are presented.

In the second part, a comprehensive study of the basic SSFA is developed including the following items: (1) redefinition of the basic SSFA as a subset of the generalized SSFA, (2) systematic presentation of the methods for formulation, i.e., the node-head method, the cotree-flow method and the mesh-flow method, and the methods for solution, i.e., the Newton-Raphson (NR) method and the Hardy Cross (HC) method, (3) characterization and evaluation of the existing methods in reference to the presented methods for formulation and solution, (4) investigation of the efficiency of NR method, HC method and their modifications in connection with the presented formulations, (5) introduction of the sparse matrix technique for solving the correction equations of NR method.

Then it is concluded that the Node-Head method linking the formulation by the node-head method and the solution by NR-B method, and the Mesh-Flow method linking the formulation by the mesh-flow method and the solution by NR-A method are most favorable for fast solution. In most real networks, since the number of meshes are smaller than that of nodes, the Mesh-Flow method takes precedence of the Node-Head method.

In the last part of this section, the most recommended linkage of methods for formulation and solution of the generalized SSFA is proposed, without giving a detailed description. Then some problems which can not be solved by the basic SSFA are solved and the effectiveness of the generalized SSFA are shown. It is also shown that the solution of the formulated problem is not unique in some cases. Therefore, a more investigation is needed on how to attain efficiently the most desirable solution in such cases.

CHAPTER 4 ANALYSIS AND DESIGN OF WATER DISTRIBUTION NETWORKS FOR IRRIGATION

4.1 Introduction

As the means of distributing irrigation water, traditional open ditch systems are in the trend of being replaced by closed conduit (pipe) systems. The reason is that loss of water through transportation is reduced, and control and maintenance of the system are facilitated by the closed system.

This chapter develops new methods for modeling and design of the irrigation-water distribution network (briefly IWDN).

It is one of the special features of IWDN that water is almost uniformly tapped at many points along service pipes in the fields. The "link outflow model (L-model)" to be proposed in the next section, Section 4.2, reflects this feature, and makes a more precise analysis and design of IWDN possible than before. As the new model is a revision and a generalization of the traditional N-model, it is also effectively applicable for the city-water distribution network (CWDN).

For optimization of the design of WDN, various methods have been proposed so far /4.1/-/4.14/, but every of them, except that for tree-shaped networks, has some weakness in practical use. This is due to the fact that design problems are usually formulated into nonlinear programming problems which generally need quite a lot of time for solution. The maximum size of the network treatable by nonlinear formulation is reported to be about 100 in links /4.11/-/4.14/.

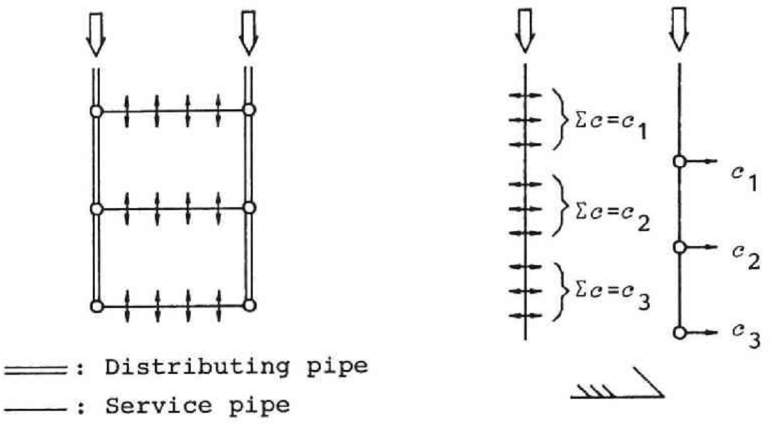
Further, in preceding works, only the cost (the cost of implementing the design and/or that of operating and maintaining the system) is introduced as the objective function while any index related to reliability (stability) of water supply has not been introduced explicitly. It goes without saying that the layout of pipes giving the minimum-cost solution is tree-shaped. Thus conventionally, from the viewpoint of reliability, upper and lower limits of pipe diameters are introduced as constraints for preventing the network to be reduced to a tree-shaped one. But it is not well-grounded how to assign proper values for those limits. Moreover, a number of local minimum solutions exist associated with a variety of tree-shaped layouts, and then decision of the most desirable solution gets difficulty.

In the third section of this chapter, Section 4.3, a design procedure consisting of two stages is proposed. The first stage determines water-flows in all the links so as to minimize an objective function which represents reliability of the water supply. The second stage is to determine pipe diameters and pump heads which minimize the total cost of pipes, pumps and the power for pumping, under the flow conditions fixed in the previous stage and the node-head conditions prescribed. Mathematically, the first stage requires solution of a set of linear simultaneous algebraic equations, and the second is to solve only a linear/separable programming problem. Hence it is quite feasible even for large-scale networks which have about 1,000 nodes and/or 1,500 links. Moreover, it is applicable to CWDN in the same manner.

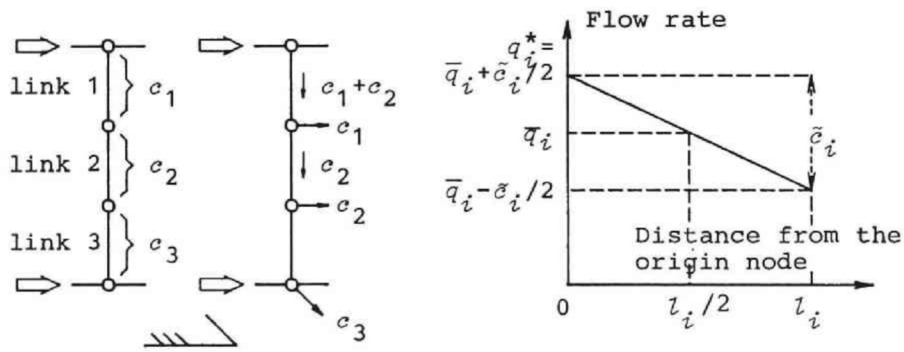
The method has been applied to networks of various sizes, and the results are quite satisfactory. Among them, typical examples are illustrated and discussed in Section 4.4.

4.2 Link Outflow Model

One of the special features of IWDN is that the pipes constituting the network are classified into two sorts as shown in Fig. 4.1(a): the distributing pipe along which no water is withdrawn and the service pipe along which water is withdrawn almost uniformly. However, in order to apply the existing methods to analysis and design, consumptions withdrawn along a service pipe are to be properly aggregated and associated with a node. In case of a single-source network, since the direction of the flow in every link can be precisely estimated, more or less, a design which ensures a sufficient water head can be made by aggregating the consumption along each link to the downstream node as shown in Fig. 4.1(b). Contrary to this, in case of a multi-source network, estimation of the direction of link-flows is difficult. In addition, since the minimum effective head in designing IWDN (usually 2-3m) is considerably lower than that for CWDN (15-20m), it is probable that water does not flow out at some sections of the service pipes if the scattered consumptions are aggregated inadequately. For example, if the consumption along every link is aggregated and associated with its downward node as shown in the left of Fig. 4.1(c), it may happen that the flow rate of link 3 becomes zero as illustrated in the right of the figure. Then a pipe of the minimum diameter is chosen for the link. Consequently, it becomes impossible to withdraw the consumption c_3 from link 3 because the head loss of the link becomes too large.



(a) Example of an IWDN (b) Aggregation of the scattered consumption



(c) Difficulty caused by the aggregation (d) Link outflow model

Fig. 4.1. Model of an irrigation-water distribution network.

To solve such difficulties, a new model called a "link outflow model (L-model)" is proposed. The model supposes that water is withdrawn quite uniformly from pipes as is shown in Fig. 4.1(d). It describes the actual state of IWDN more adequately than the N-model, therefore, it improves much accuracy of the analysis and the design of IWDN.

In the following, fundamental equations of the L-model, i.e., the link characteristics, the flow conservation law, and the head difference loop law are introduced.

4.2.1 Characteristics of pipe links

Suppose that the Hazen-Williams formula holds for the relationship between the flow rate and the head difference of the pipe link. Then, the characteristic equation of pipe link i from which water is uniformly withdrawn is given by

$$\begin{aligned} h_i &= \int_0^{l_i} \bar{r}_i (q_i^* - \tilde{c}_i x/l_i)\,|q_i^* - \tilde{c}_i x/l_i|^{0.85} dx \\ &= \bar{r}_i l_i (|q_i^*|^{2.85} - |\tilde{c}_i - q_i^*|^{2.85})/2.85\tilde{c}_i \qquad (4.1) \\ \bar{r}_i &\triangleq 10.666 CH_i^{-1.85} d_i^{-4.87} \end{aligned}$$

where

q_i^* : flow rate at the origin node of link i (positive for inflow) $[m^3/s]$

$\tilde{c}_i$: sum of the consumption withdrawn from link i $[m^3/s]$

The other symbols are identical with those defined in the previous chapters. If $\tilde{c}_i > q_i^* > 0$, then water flows into the link from its both ends, and the head takes the lowest value at $x_m \triangleq l_i q_i^*/\tilde{c}_i$. If $\tilde{c}_i = 0$, then the traditional relation

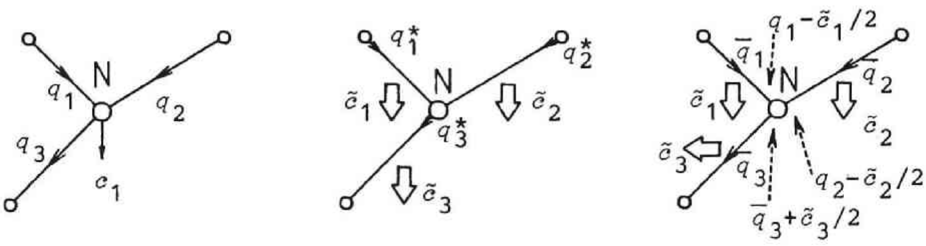
$$h_i = \bar{r}_i l_i q_i^* |q_i^*|^{0.85} \qquad (4.2)$$

holds.

4.2.2 Flow conservation law (FCL)

In the N-model of Fig. 4.2(a), FCL at point N is described by

$$q_1 + q_2 - q_3 - c_1 = 0 \qquad (4.3)$$



(a) N-model (b) L-model (c) L-model

Fig. 4.2. Flow conservation law.

while in the L-model of Fig. 4.2(b), it is described by

$$\begin{aligned} &q_1^*-\tilde{c}_1+q_2^*-\tilde{c}_2-q_3^* \\ &\quad = q_1^*+q_2^*-q_3^*-(\tilde{c}_1+\tilde{c}_2) = 0 \end{aligned} \tag{4.3)'$$

From comparison of Eqs. (4.3) and (4.3)', it can easily be seen that FCL of the L-model is equivalent to that of the N-model (called N*-model) which is derived from the L-model by the following procedure: Aggregate each $\tilde{c}_i$ to the terminal node of link i and let the flow rate q_i^* be the constant flow rate of link i. The flow adopted such like q_i^* is called a representative flow rate. This procedure is also valid for the case where both link outflow (i.e., scattered consumption) and nodal outflow (i.e., concentrated consumption) exist.

Alternatively, $\bar{q}_i$, i.e., the flow rate at the middle point of link i can be made a representative flow rate of the link. For that, derive the equivalent N-model (called $\bar{\text{N}}$-model) by aggregating the halves of $\tilde{c}_i$ to both ends of link i. See Fig. 4.2(c).

The flow rate of in/outflow link i in the N* or the $\bar{\text{N}}$-model is denoted by c_i.

In design problems, since all the supply rates from sources and the consumptions are given usually, all the c_i's are known quantities. Hence, FCL with respect to $\overline{q}_i$'s can be written with use of the datum-node reduced incidence matrix as follows:

$$\begin{array}{c} 1 \\ n-1 \end{array}\left[\begin{array}{cccc} 1 & A_{0t} & A_{0c} & 0 \\ 0 & A_{jt} & A_{jc} & I_j \end{array}\right]\left[\begin{array}{c} c_0 \\ \overline{Q}_t \\ \hline \overline{Q}_c \\ C_j \end{array}\right] = 0 \qquad \begin{array}{l} \text{Tree} \\ \\ \text{Cotree} \end{array} \tag{4.4}$$

$$\begin{array}{cccc} 1 & n-1 & m-n+1 & n-1 \end{array}$$

Incidence matrix

where $\overline{Q}_t$, an $(n-1)$ vector, denotes the flow rates at the middle point of the pipe links of a tree, $\overline{Q}_c$, an $(m-n+1)$ vector, those of the cotree. An arbitrarily chosen in/outflow link is included in the tree, and its flow rate is denoted by c_0, which is treated as an unknown quantity (refer to Section 2.5). The other in/outflow links whose flow rates are denoted by C_j, an $(n-1)$ vector, are included in the cotree. The size of each submatrix is shown outside of the matrix.

From the second row of Eq. (4.4), we obtain

$$\overline{Q}_t = -\Gamma_{tc}\overline{Q}_c - \Gamma_{tj}C_j \triangleq -A_{jt}^{-1}(A_{jc}\overline{Q}_c + C_j) \tag{4.5}$$

4.2.3 Head difference loop law (HDLL)

HDLL is identical with that for the N-model and can be written as:

$$\begin{array}{c} m-n+s \\ n-s \end{array}\left[\begin{array}{cccc} B_{cs} & B_{ct} & I_c & 0 \\ B_{js} & B_{jt} & 0 & I_j \end{array}\right]\left[\begin{array}{c} H_s \\ H_t \\ \hline H_c \\ H_j \end{array}\right] = 0 \qquad \begin{array}{l} \text{Tree} \\ \\ \text{Cotree} \end{array} \tag{4.6}$$

$$\begin{array}{cccc} s & n-s & m-n+s & n-s \end{array}$$

Fundamental tieset matrix

The vector H of head differences are partitioned into four sub-vectors, $H_{\cdot}$. The subscript s denotes source links, t pipe links in a tree, c pipe links in the corresponding cotree, j consumption links. s also denotes the number of source links.

The subsets t, c and j in Eq. (4.4) are different from those in Eq. (4.6) in general. However, since Eqs. (4.4) and (4.6) are never used simultaneously, the symbols of t, c and j are commonly used in the both equations.

4.3 Optimal Design of Water Distribution Networks

The design procedure presented here starts with situation that a layout of the network is already prescribed. Actually, the layout is fixed, more or less, by topographical conditions and/or existing facilities such as roads. If there are several alternative plans for the layout, the following procedure is applied to each of them and the most desirable one is finally adopted.

4.3.1 Allocation of water-flows

The first stage of the design is the allocation of water-flows to links according to the following principle:

$$\text{(P1)} \quad \underset{\bar{q}_i}{\text{minimize}} \quad F_1 \triangleq \sum_{i=1}^{m} l_i \bar{q}_i^{\,2} \tag{4.7}$$

Since the flow rate may spacially vary even in a link of the L-model, $\bar{q}_i$ is adopted as the representative value of the flow rate in link i. (P1) implies that the water is to be delivered to consumers through as short paths as possible, and at the same time through as many paths as possible in a distributed way. A greater advantage is that (P1) can be applied to multi-source networks as well as to single-source networks. As F_1 is a quadratic of $\bar{q}_i$'s, it is favorable also for mathematical treatment. Furthermore, (P1) can easily be learned by intuition.

Takakuwa has proposed the following principle:

$$\text{(P0)}\quad \underset{q_i}{\text{minimize}} \quad F_0 \triangleq \sum_{i=1}^{m} q_i^{\,2} \tag{4.7)'$$

for allocation of water-flows /4.15/. He has shown that the use of (P0) improves generally the supply reliability of the network.

However, (P0) has not been practically used mainly due to the following reasons:

(1) The length of pipe links can not be reflected.

(2) In multi-source networks as well as in single-source networks, the inflow rates from sources are specified generally in design. Whereas those flow rates can not be fixed to specified values by the method of Takakuwa: He puts all the source links in a tree in Eq. (4.4).

(3) The pattern of water-flows obtained by (P0) is usually quite different from that desirable from practical viewpoints as will be discussed in the following.

(P1) and (P2), a modification of (P1) presented later, will solve these problems.

As for tree-shaped networks, if a pipe breaks at a point, it becomes quite impossible to supply water to the downstream area of the point. In contrast, looped networks whose water-flows are determined by solving (P1), even if a trouble occurs at a point, can keep up the water supply to the downstream area through some other paths, to some extent, so far as the layout is reasonably planned. In this sense, it can be said that (P1) guarantees fairly high reliability (stability) of the supply on the network.

By the use of Eq. (4.5), all the $\overline{q}_i$'s are denoted by $\overline{Q}_c$. Therefore the solution of (P1) is given by solving $\partial F_1/\partial \overline{Q}_c=0$, i.e., by solving the following linear simultaneous equations:

$$
\begin{aligned}
&\begin{bmatrix} -\Gamma_{tc}^T & I \end{bmatrix} \mathrm{diag}(l_i) \begin{bmatrix} -\Gamma_{tc} \\ I \end{bmatrix} \overline{Q}_c \\
&\qquad = \begin{bmatrix} -\Gamma_{tc}^T & I \end{bmatrix} \mathrm{diag}(l_i) \begin{bmatrix} \Gamma_{tj} \\ 0 \end{bmatrix}
\end{aligned}
\tag{4.8}
$$

Since the coefficient matrix of $\overline{Q}_c$ in Eq. (4.8) is positive definite, the solution of Eq. (4.8) is uniquely determined.

(P1) is desirable from the viewpoint of reliability. From a practical viewpoint, however, it is sometimes desired, by some reasons, to concentrate water-flows to some links. For example, the flow pattern must be adapted to the layout of roads and/or the topography of the field; under wide roads large (major) pipes with high flow-rate are to be buried, and under narrow roads small (minor) pipes with low flow-rate. In addition, the cost can be reduced if the flow pattern is adjusted close to tree-shaped one, which will be shown in 4.4.2. In order to fit the flow pattern as desired, (P1) is modified as follows:

$$
\text{(P2)} \quad \underset{\overline{q}_i}{\text{minimize}} \quad F_2 \triangleq \textstyle\sum_{i=1}^{m} w_i^{-1} l_i \overline{q}_i^{\,2} \tag{4.9}
$$

Namely, a weight parameter w_i is introduced to each link. The w_i's of the major links desired to have high flow-rate are given greater values than 1 according to requirement of concentrating the flow, and those of the other links 1. In practice, a search is necessary for finding out a most desirable flow pattern by changing the values of w_i's and looking at resulted flow patterns.

It is noted that the maximum allowable flow rate for each link might be introduced explicitly as a constraint of (P1). But, by so doing, the solution process becomes far more complicated and time consuming.

4.3.2 Minimum-cost design

Consider to select one or two kinds of "commercially available pipes" for each link so as to minimize the total pipe cost, given the heads at all sources, the flow rates in all links, and constraints on the heads at demand nodes. This problem can be cast as a linear program (LP) as was shown by Karmeli /4.1/ and Kally /4.2/ for the N-model.

In the following, the method (hereafter called the "commercial-pipe LP method") is presented and then its applicability to the L-model is discussed.

(a) Commercial-pipe linear-programming method

Let the list of all the commercially available diameters be given by $\{d_1, d_2, \ldots, d_\mu, \ldots\}$, and suppose that each link consists of one or more segments, each having one of those diameters, as shown in Fig. 4.3. In addition, let the length of the segment having the μ-th diameter in link i be denoted by $x_{i\mu}$. Then

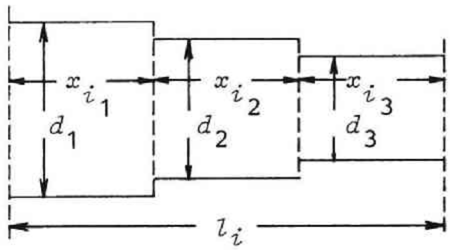


Fig. 4.3. Structure of link i.

$$\sum_{\mu=\mu_{i_1}}^{\mu_{i_\psi}} x_{i\mu} = l_i \qquad (i = 1, 2, \ldots, m)$$

(Condition on the segment length) (4.10)

holds. The list of candidate diameters $(d_{\mu_{i_1}}, \ldots, d_{\mu_{i_\psi}})$ may be different by link, and the size of the list is generally limited to the range of 2 to 8 by restriction on the speed of water flow. The head loss in link i composed of the segments as above is given by

$$h_i = \sum_{\mu=\mu_{i_1}}^{\mu_{i_\psi}} \bar{r}_\mu x_{i\mu} q_i |q_i|^{0.85} \qquad \text{(Link characteristics)} \qquad (4.11)$$

where q_i is the constant flow rate in link i.

The consolidated capital cost of pipe link i, including the cost of laying, labor, etc., is given by

$$\Pi_i = \sum_{\mu=\mu_{i_1}}^{\mu_{i_\psi}} \pi_\mu x_{i\mu} \qquad \text{(Link cost)} \qquad (4.12)$$

where π_μ is the consolidated capital cost per unit length of the pipe having the μ-th diameter. Therefore the total cost related to the pipe is given by

$$S = \Sigma_{i=1}^{m} \Pi_i \quad \text{(Total cost)} \tag{4.13}$$

On one hand, by use of Eq. (4.6), HDLL and the node-head condition are given as follows:

$$B_{cs}H_s + B_{ct}H_t + H_c = 0 \qquad \text{(HDLL)} \tag{4.14}$$

$$\underline{H}_j \leqq H_j = -B_{js}H_s - B_{jt}H_t \leqq \overline{H}_j \qquad \text{(Node-head condition)} \tag{4.15}$$

where $\underline{H}_j$ and $\overline{H}_j$ are the vectors denoting the lowest and the highest admissible heads at nodes. Eq. (4.14) may be replaced by Eq. (3.13).

Now, the problem is to minimize the total cost, Eq. (4.13), subject to the constraints, Eqs. (4.10), (4.14) and (4.15). By substituting Eq. (4.11) into Eqs. (4.14) and (4.15), since $\overline{r}_\mu$ and q_i are known, we obtain a linear program whose unknowns are the length of the segments, $x_{i\mu}$'s.

In the optimal solution of this linear program, each link consists of either a single segment or two consecutive ones in the candidate list. This is because π_μ is proportional to 1.3 to 2nd power of the diameter.

When pumps are introduced to sources, the sum of the pipe cost and the pump cost (including the capital, the operation and the maintenance costs) should be minimized. The flow rates of sources are usually given. And for a constant flow rate, such a relation as shown in Fig. 4.4 holds between the pump head and the pump cost. Therefore, we incorporate the pump head p_{si} of source s_i into the decision variables and substitute

$$h_{si} + p_{si} \tag{4.16}$$

for h_{si} in Eqs. (4.14) and (4.15), and further, add the pump cost

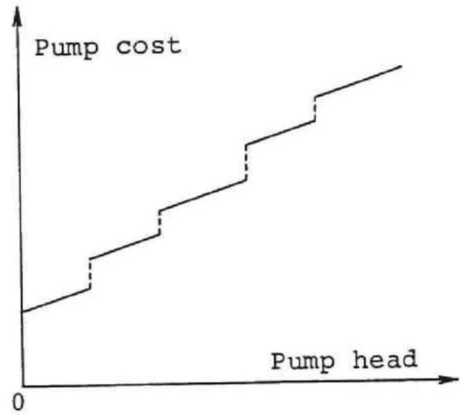


Fig. 4.4. Relationship between the pump head and the pump cost (for constant flow rate).

$$\pi_{si} = \pi_{si}(p_{si}) \tag{4.17}$$

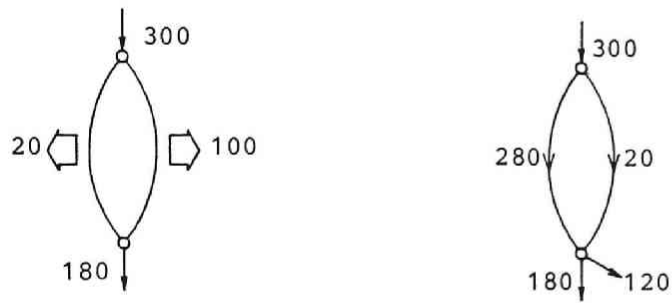
to the pipe cost of Eq. (4.13). As a result, we obtain a separable programming problem.

(b) Application to the link outflow model

In the L-model, since the flow rate is not constant by place in the link, we adopt the following quantities for q_i in Eq. (4.11).

<u>A-method</u> We use the maximum flow rate in link i for q_i in Eq. (4.11). If water flows into the link from both of its ends, divide the link into two sublinks by introducing a new node at the most downstream point and then use the flow rate at each end for q_i of each sublinks.

This procedure is considered to be an extended and more precise version of the traditional aggregation of scattered consumptions. For example, assume that the scattered consumption on a link is aggregated correctly to its downstream node as is shown in Fig. 4.5(a). If this aggregated quantity is considered



(a) Consumptions along links (b) Improper allocation of water-flows

Fig. 4.5. Aggregation of scattered consumptions.

to be a mere nodal outflow, such an allocation as shown in Fig. 4.5(b) is possible. Note that, in this allocation, the necessary quantity can not be withdrawn from the right link.

A-method is quite free from this sort of difficulty. In the design of IWDN, since the minimum head is relatively low, a moderate over-estimation of the head loss in each link is considered quite favorable from the practical viewpoint.

B-method First, if water flows into the link from both of its ends, divide the link into two in the like manner as A-method. Second, assume that each link is constant in its diameter, and find the constant flow rate $\tilde{q}_i$ which gives the same head loss caused by q_i^* and $\tilde{c}_i$ in the link. Namely, determine $\tilde{q}_i$ by solving

$$\bar{r}_i l_i (|q_i^*|^{2.85} - |\tilde{c}_i - q_i^*|^{2.85})/2.85\tilde{c}_i = \bar{r}_i l_i \tilde{q}_i |\tilde{q}_i|^{0.85} \quad (4.18)$$

Then $\tilde{q}_i$ is obtained explicitly to be

$$\tilde{q}_i = \rho|\rho|^{-0.85/1.85},$$
$$\rho \triangleq (|q_i^*|^{2.85} - |\tilde{c}_i - q_i^*|^{2.85})/2.85\tilde{c}_i \quad (4.19)$$

and it is used for q_i in Eq. (4.11).

Now, let h_i^{opt} be the head loss of link i in the optimal solution of LP problem formulated by using $\tilde{q}_i$'s. If link i includes only a kind of segment in the optimal solution, h_i^*, i.e., the head loss caused in the link by q_i^* and $\tilde{c}_i$, is equal to h_i^{opt}. On the other hand, if link i includes two kinds of segments whose diameters are d_{i_1} and d_{i_2}, it can be easily proved that $h_i^*<h_i^{opt}$. In the latter case, the lengths of the segments are recalculated so that h_i^* becomes equal to h_i^{opt} by solving the following equation whose unknown is x:

$$h_i^{opt} = l_i[\bar{r}_{i_1}\{(q_i^*)^{2.85}-(q_i^*-\tilde{c}_i x/l_i)^{2.85}\} + \bar{r}_{i_2}\{(q_i^*-\tilde{c}_i x/l_i)^{2.85}-(q_i^*-\tilde{c}_i)^{2.85}\}]/2.85\tilde{c}_i \tag{4.20}$$

The length of the segment whose diameter is d_{i_1} is given by x, and that of d_{i_2} by (l_i-x). We put the notations so that $d_{i_1}>d_{i_2}$ and the direction of the link is set so that the relation $q_i^*\geq\tilde{c}_i\geq 0$ holds.

As mentioned above, B-method evaluates the head loss of each link exactly and its solution is sufficiently close to the absolute minimum-cost solution satisfying all the constraints.

(c) Rounding of segment length

By our design procedure, a link may finally be composed of more than one (the maximum is four) segments. That is to say, in the stage of determining q_i's in Eq. (4.11) by A- or B-method, a link is divided into two sublinks if water flows into the link from both of its ends, and each link/sublink may be composed of two segments in the optimal LP solution. Consequently some segments are probable to be found quite short. From practical viewpoint, however, use of such short segments is undesirable, because it requires additional cost of compromise joints. Additionally, the length of each segment is desired to be an integer multiple of the length of the commercially available pipe (usually 5m), in order to save the cost for cutting the pipe. As a result of the above discussion, pipes composing a link are determined practically according to the following rules. First, consider the case that a link is composed of two segments.

1) Omit the residual of the length of the smaller diameter segment divided by 5m, while raise that of the larger diameter one to 5m.

2) If the smaller diameter segment occupies more than 80% (can be changed) of the link, adopt the smaller diameter for the whole link; otherwise adopt the larger one.

3) If one of the diameters is equal to that of a neighboring link and if no other link branches off from the interconnecting node, shift the node itself, if possible, as shown in Fig. 4.6.

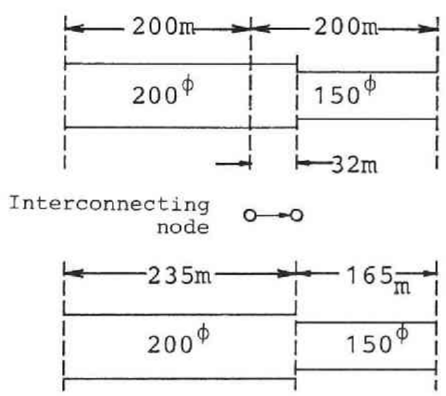


Fig. 4.6. Shift of the interconnecting node and rounding of segment length.

Second, if a link is composed of more than two segments, apply the above rules to each sublink given in the stage of determining q_i's in Eq. (4.11) and then apply the rules again to the whole link.

4.4 Examples

4.4.1 Fundamental study by a small network

The procedure presented in the preceding section is applied to the design of the single-source network of Fig. 4.7. No pump is introduced. Typical results are summarized in Table 4.1. Table 4.1 also shows the results of the design by use of a typical existing method (M1) /4.16/,/4.17/.

M1 is originally devised for the design of single-source networks. It requires, at the outset of the design, a designer to aggregate the scattered consumption along each link of service pipes to its downstream node, and to determine the flow rate in each link as shown in Fig. 4.8. Figure 4.8(a) shows the case where one end of the link can evidently be identified as the upperstream one and another end the downstream one, while (b) shows the case where both ends of the link are

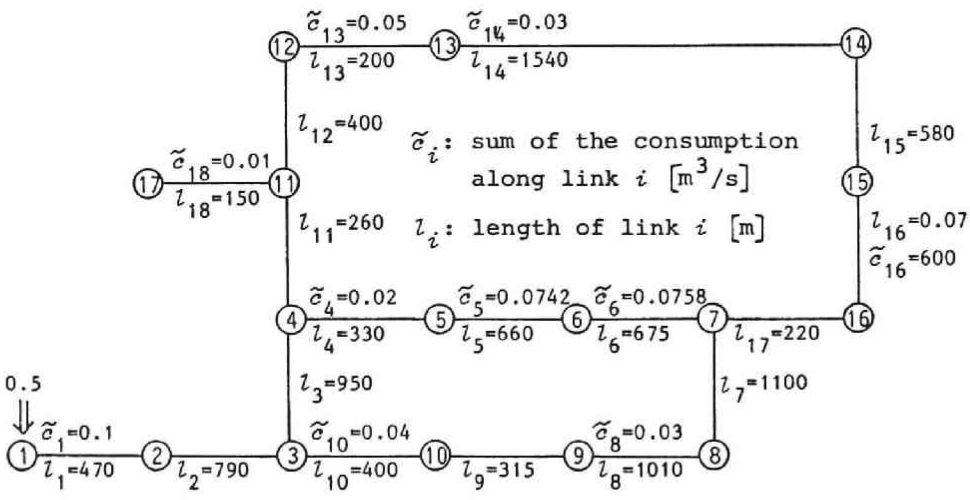


Fig. 4.7. Network for design example.

Table 4.1 Results of several designs

Method of design	M1	M2 *1	M3	M4 *1	M5
Allocation of water-flows	By experience	min $\Sigma w l q^{2}$	min $\Sigma l q^{-2}$	min $\Sigma w l q^{-2}$	min $\Sigma l q^{-2}$
Aggregation of the consumption	By experience		A-method		B-method
Minimum-cost design	Equal head- *3 slope method	Commercial-pipe LP method			
Cost	427,517kg	405,222kg	411,077kg	421,565kg	385,391kg
The minimum *2 head	(L6) 9.30m (L14) 4.35m	(L6) < 0 (L16) 0.33m	(L6) 7.84m (L16) 5.05m	*4 (L16) 5.62m	(L6) 3.00m (L16) 3.00m
Pattern *2 change	(L6) 8.33m (L14) < 0		(L6) 6.92m (L16) 3.86m	*4 (L15) 4.27m	(L6) 2.67m (L16) 0.98m

*1 In M2 and M4, w_i (i = 3, 4, 5, 6) are supposed to be 1.667, and others to be 1.0.

*2 The link in the parenthesis shows that the (locally) minimum head appears at a point on the link. The ground level at each point on the link is interpolated by use of those at the both ends of the link.

*3 No optimization technique is explicitly used.

*4 No locally minimum value appears along link 6.

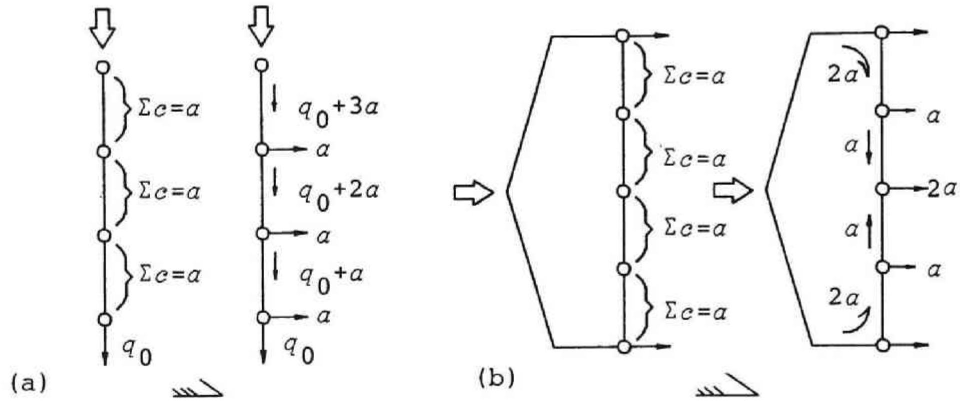


Fig. 4.8. Examples of aggregation of scattered consumptions and determination of link-flows in M1.

located at equal distance from the source. It is recommended that a layout of the network is planned so that every section of service pipes is located as shown in (b) for attaining high reliability of water-supply. For the flow pattern determined as above, M1 chooses the diameter of a pipe so that the relation between the head loss and the distance from the source is made linear as far as possible.

The method M1 has been used successfully in a lot of practical designs, and has gained a high appraisal. It is to be noted that the concept of reliability in M1 is inherited, in a modified manner, to our method. In spite of its various advantages, M1 has some serious drawbacks: The designer has to aggregate the scattered consumption, and moreover has to decompose the network into some single-source networks prior to the aggregation, if it includes more than one source. Hence quality of the network designed sharply depends on skill of the designer. An example of aggregation of the consumption, i.e.,

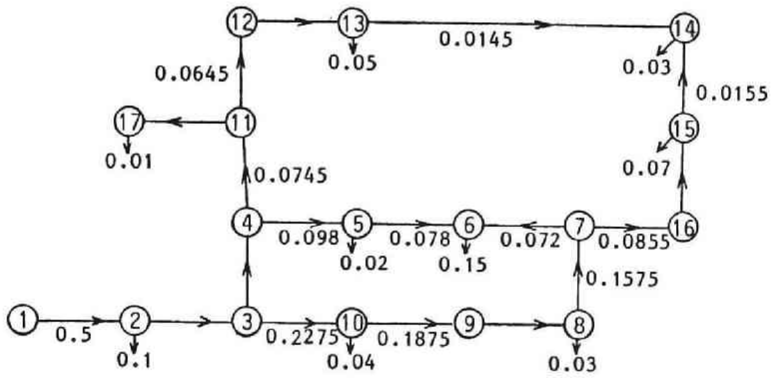


Fig. 4.9. N-model and allocated water-flows in M1.

the N-model of the network of Fig. 4.7, along with the allocated water-flows, is shown in Fig. 4.9.

In the present study, the following varieties of the procedure are examined and compared with one another:

(M2) M2 performs the design on the N-model of Fig. 4.9. It allocates water-flows to links by solving (P2), specifying some links as major (trunk) links, and then proceeds to the minimum-cost design by using the commercial-pipe LP method.

(M3) M3 allocates water-flows to links by solving (P1), and then solves the commercial-pipe LP problem by using A-method to determine the constant flow rate of each link.

(M4) M4 allocates water-flows to links by solving (P2) with the same specification of trunk links as of M2, and solves the commercial-pipe LP problem by using A-method to determine the constant flow rate of each link.

(M5) M5 allocates water-flows to links by solving (P1), and solves the commercial-pipe LP problem by using B-method to determine the constant flow rate of each link.

In Table 4.1, the total cost is shown in terms of the total weight of the required pipes, which is almost proportional to the cost. The table also shows the results of the steady-state flow analysis performed on each designed network, for the consumption pattern of Fig. 4.7 (PAT1). The (locally) minimum value of the head and the corresponding point/link are shown in the column of "minimum head".

When A-method is adopted in the commercial-pipe LP problem, since the head loss in each link is estimated a little greater than the real value, the minimum head produced by PAT1 is generally higher than the minimum admissible value (2m).

The designed networks are also tested for another pattern of the consumption (PAT2). The consumptions of links 13 and 15 are changed from 0.05 to $0.0 m^3/s$ and from 0.03 to $0.08 m^3/s$, respectively. The minimum value of the head for PAT2 is shown in the column of "pattern change" in Table 4.1.

Now, we examine the results of M1 to M5 in detail.

(a) Comparison with the existing method

(Effectiveness of the L-model)

A comparison between M2 (N-model) and M4 (L-model) proves the effectiveness of the L-model. Figure 4.10 shows the allocated flows and the selected diameters around node 6 by M2.

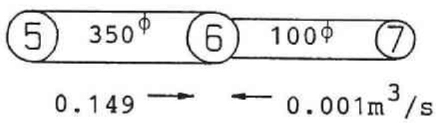


Fig. 4.10. Allocated water-flows and selected diameters around node-6.

Most of the consumption from node 6 is supplied from the side of node 5, and the flow rate in link 6 is quite small. Accordingly, the diameter of link 6 is designed small and then $\tilde{c}_6$ can not be withdrawn from the link (i.e., the head becomes negative along the link). That is, the trouble caused by the aggregation of the consumption is disclosed straightforwardly. In contrast, by M4, the head is sufficient along the same link.

(b) Comparison with the existing method M1 (Cost and stability)

Let us compare M3 with M1. Although the cost by M3 is lower than that by M1, the minimum head by M3 is 5.05m and is higher than that by M1, 4.35m. In addition, for PAT2, sufficient head is obtained by M3, while the head becomes negative along link 14 by M1. For some different patterns of the consumption, M3 generally presents better results than M1 does.

The above discussions of (a) and (b) verify that the proposed procedure gives fairly better results than M1. It is due to the fact that our procedure uses more precise model than that of M1, establishes a concrete index for ensuring reliability and utilizes sound mathematical analysis and programming techniques.

(c) Effect of the concentration of water-flows

Now let us compare M4 with M3. The cost by M4 is higher than that by M3. In designing a real network, however, since we usually arrange trunk links in a tree-like form, the cost decreases in general by concentration of water-flows. The specification of trunks adopted in M4 is an exceptional one for verifying the effectiveness of the L-model, and causes a slight detour in water distribution, producing higher cost.

Concerning the minimum head for PAT1 and PAT2, the difference is 5.05-3.86=1.19m in M3, and 5.62-4.27=1.35m in M4. That is, the difference in M4 is a little greater than that in M3. It is a general tendency that the network of tree-shaped trunks produces a greater difference in the minimum head than the network designed by M3, when the consumption pattern is changed slightly from that used for the design.

(d) Features of B-method

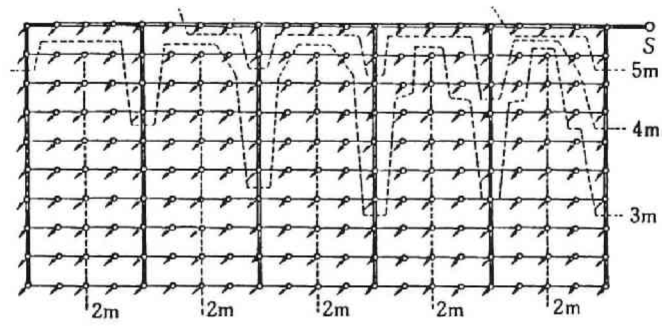
B-method for determining the constant flow rate of each link in the commercial-pipe LP problem evaluates the head loss of each link strictly. Therefore, the cost by M5 is sharply cut down from that by M3 and the minimum head becomes exactly equal to the minimum admissible value.

In order to give allowance for design, it is recommended practically (1) to use A-method and overestimate the head loss in each link, or (2) to use B-method with a minimum admissible head a little higher than required.

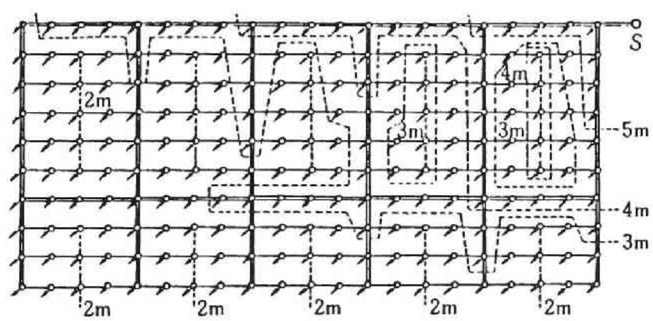
4.4.2 Example of designing a large-scale network

For the single-source network of 211 nodes and 255 links of Fig. 4.11, several patterns of water-flows are determined by specifying various trunk links, and the commercial-diameter LP problem is solved for each of them. The head at the source (denoted by s in the figure) is supposed to be 5.72m. The ground level is supposed to be 0m everywhere, and every consumption is 0.005m^3/s. Figure 4.11 shows typical results. The segment length is not rounded.

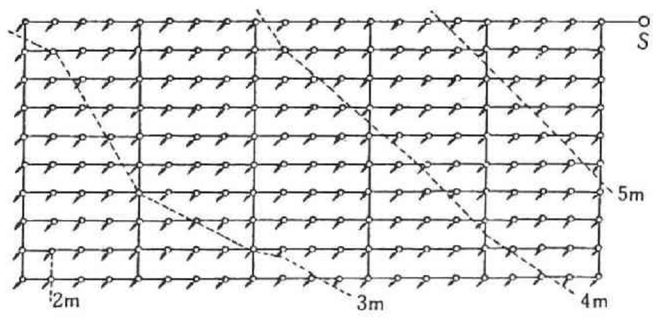
The cost is minimum in (a) and maximum in (c). On one



(a) Tree-like trunk links ($183,986 \times 10^3$ Yen)



(b) Looped trunk links ($204,365 \times 10^3$ Yen)



(c) No trunk link ($244,015 \times 10^3$ Yen)

Fig. 4.11. Examples of designing a large-scale network (cost in Yen).

hand, as shown by the contours in Fig. 4.11, the head is 2m (lowest) at the every middle point of non-trunk section consisting of four links in (a), while at only two points in (c). The head is relatively high all over the network in (c). Therefore, if a breakdown occurs at a point on a trunk link, the water-supply to its downstream nodes is stopped in (a), while the supply is kept almost unchanged except for the case the point is quite close to the source. In (b), since the trunk links form loops, the supply through another trunk route is possible to some extent.

This example shows explicitly the trade-off between the reduction of the cost and the enhancement of the supply reliability. In a way like this, the present method can afford a design which reflects fully the requirement or preference of the user.

4.5 Concluding Remarks

In this chapter, new methods for modeling and design of irrigation-water distribution networks have been proposed.

First, in Section 4.2, the idea of the link outflow model has been introduced. The model improves much the accuracy of analysis and design of the irrigation-water distribution network. It is effective also for the city-water distribution network.

Second, in Section 4.3, a method for design consisting of two stages has been introduced. In Section 4.4, the features of the method have been examined from several viewpoints in applications to various networks. The prominent features of the method are summarized as follows; (1) it enables us to consider explicitly the trade-off between enhancement of the supply reliability and reduction of the construction cost, (2) it is applicable to the multi-source network as well as to the single-source network, (3) it requires solution of merely a set of simultaneous algebraic equations and a linear/separable programming problem, then it is quite feasible even for large-scale networks which have about 1,000 nodes, 1,500 links and/or 300 loops.

The method has already been used for practical design of large-scale networks, and has proved its excellent capability compared with conventional techniques which solve nonlinear programs for minimizing the cost without paying attention to the supply reliability.

By the way, the existing methods for the design of the water distribution network, including the presented one, are those usable for a newly-constructed network. But nowadays in Japan and in many other developed countries, most of the city-water distribution networks are constructed by enlarging and/or partially replacing the old ones. The present method can be extended to cope with the problem for enlarging an existing network /4.18/.

CHAPTER 5 PLANNING OF PRESSURE CONTROL IN WATER DISTRIBUTION NETWORKS INCLUDING LOCATION OF CONTROL POINTS

5.1 Introduction

The uniform pressure control in a water distribution network, i.e., keeping the water head within a certain range at any point in the distribution network, is strongly desirable in order to guarantee smooth water supply, to decrease leakage and waste of water, and to protect pipes, etc. Since the heads in the network considerably change as the consumptions change, we have to apply a control to the network by means of pumps and valves for regulating the heads.

This chapter is concerned with the uniform pressure control scheme to find out a proper way for operations of pumps and valves, prescribed with the water levels in reservoirs and the consumptions at all demand nodes. Such a study under the steady-state condition is indispensable especially in the planning stage, and gives us much useful knowledge even for the on-line control to cope with fluctuations of consumptions. In the past works /5.1/-/5.3/, it is premised that control points, i.e., the locations of pumps and valves are already established. In contrast, the present study tries to find out their desirable locations as well as their operation schemes. This is because rational establishment of control points improves the efficiency of the control by a large margin in general.

The problem is formulated into a nonlinear programming problem, and then it is solved by iterating solutions of a chain of linear programs. Due to the linearization, the solution procedure is endowed with an excellent applicability even to large-scale networks.

In Section 5.2, an optimization problem is formulated. Section 5.3 develops a basic algorithm of the solution. Section 5.4 points out some computational difficulties which arise in application of the basic algorithm, together with a key cause of the difficulties. In Section 5.5, a revised algorithm which ensures the convergence to the optimal solution is presented. In Section 5.6, the revised algorithm is applied to some networks of practical size, and their important results are shown.

5.2 Formulation

A pressure control planning problem including location of control points is formulated in this section. At the outset, fundamental equations are presented in the forms adjusted to this problem.

5.2.1 Fundamental equations

(a) Principal variables

Table 5.1. Principal variables of WDN

	Number of links	Vector of head differences	Vector of Flow rates
Source link	s	P_e (known)	C_e (unknown)
Consumption link	$n - s$	P_j (unknown)	C_j (known)
Conduit-element link (Pipe link)	m	$H_k=[h_j]$ (unknown)	$Q_k=[q_j]$ (unknown)

The principal variables and their sizes are listed in Table 5.1.

(b) Characteristic equation of the pipe link

Suppose that the Hazen-Williams formula holds for the relationship between the flow rate and the head loss of the pipe link, and denote a pressure gap given by a pump or a valve by z. Then, the characteristic equation of pipe link i equipped with a pump or a valve is written as

$$h_i = r_i q_i |q_i|^{0.85} - z_i \tag{5.1}$$

r_i is the resistance factor defined in Eq. (2.1).

Every pipe link is taken to be a candidate where a pump or a valve is introduced. If $q_i z_i > 0$, z_i denotes the pressure gap given by the pump, while if $q_i z_i < 0$, that by the valve.

(c) Flow conservation law (FCL)

FCL can be written as:

$$\begin{array}{c} s \\ n-s \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} I_e & A_{et} & A_{ec} & 0 \\ 0 & A_{jt} & A_{jc} & I_j \end{bmatrix} \\ \begin{array}{cccc} s & n-s & m-n+s & n-s \end{array} \end{array} \begin{bmatrix} C_e \\ Q_t \\ \hline Q_c \\ C_j \end{bmatrix} = 0 \begin{array}{l} \text{Tree} \\ ---- \\ \text{Cotree} \end{array} \tag{5.2}$$

The coefficient matrix in Eq. (5.2) is a datum-node reduced incidence matrix written in a partitioned form. All the source links are included in a tree, consumption links in the corresponding cotree. Q_t, an $(n-s)$ vector, denotes the flow rates of the pipe links of the tree, while Q_c, an $(m-n+s)$ vector, those of the cotree. The size of each submatrix is shown outside the matrix. From the second row of Eq. (5.2), as submatrix A_{jt} is not singular, we can obtain

$$Q_t = (A_{jt})^{-1}(A_{jc}Q_c + C_j) \tag{5.3}$$

Then it is obvious that Q_c is a set of necessary and sufficient variables to describe the flow rates of all links in the network.

(d) Head difference loop law (HDLL)

HDLL can be written as:

$$\begin{array}{cc} m-n+s \\ n-s \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{cccc} B_{ce} & B_{ct} & I_c & 0 \\ B_{je} & B_{jt} & 0 & I_j \end{array}\right] \\ \begin{array}{cccc} s & n-s & m-n+s & n-s \end{array} \end{array} \left[\begin{array}{c} P_e \\ H_t \\ \hline H_c \\ P_j \end{array}\right] \begin{array}{l} \text{Tree} \\ = 0 \text{ ----} \\ \text{Cotree} \end{array} \tag{5.4}$$

The coefficient matrix of Eq. (5.4) is a fundamental tieset matrix in a partitioned form. H_t, an $(n-s)$ vector, and H_c, an $(m-n+s)$ vector, denote the head differences of the pipe links of the same tree and cotree as used for Eq. (5.2), respectively.

5.2.2 Formulation

First, we consider to minimize the total energy, or equivalently the cost, expended in pumping while keeping the water heads at all nodes in an allowable range. This problem is formulated as follows:

(P1) minimize $f = \Sigma_{i=1}^{m}(q_i z_i + |q_i z_i|)/2$ (Pumping cost) (5.5)

subject to

$L = (l_\phi) \triangleq B_{ce}P_e + B_{ct}H_t + H_c = 0$ (HDLL) (5.6)

$\underline{P}_j \leq P_j = -B_{je}P_e - B_{jt}H_t$

$P_j \leq \overline{P}_j$ (Node-head condition) (5.7)

where $\underline{P}_j = (\underline{p}_\mu)$ and $\overline{P}_j = (\overline{p}_\mu)$ are the vectors denoting the lowest and the highest allowable values of $P_j = (p_\mu)$. Owing to Eqs.

(5.1) and (5.3), Eqs. (5.5) through (5.7) are described only by Q_c and $Z=(z_i)$. It is noted that each component of L and P_j is the sum of a linear function of Z and a nonlinear function of Q_c.

By way of example, the problem (P1) is formulated as follows for Network-1 of Fig. 5.1.

$$\begin{aligned}\text{minimize}\quad & q_1z_1+|q_1z_1|+q_2z_2+|q_2z_2|+(2-q_1-q_2)z_3\\ & +|(2-q_1-q_2)z_3|+(1-q_2)z_4+|(1-q_2)z_4|\\ \text{subject to}\quad & r_1q_1|q_1|^{0.85}-z_1-r_3(2-q_1-q_2)|2-q_1-q_2|^{0.85}+z_3=0\\ & r_2q_2|q_2|^{0.85}-z_2-r_4(1-q_2)|1-q_2|^{0.85}+z_4\\ & \qquad -r_3(2-q_1-q_2)|2-q_1-q_2|^{0.85}+z_3=0\\ & \underline{p}_1\leqq p_1=p_0-r_1q_1|q_1|^{0.85}+z_1\leqq\overline{p}_1\\ & \underline{p}_2\leqq p_2=p_0-r_2q_2|q_2|^{0.85}+z_2\leqq\overline{p}_2\end{aligned}$$

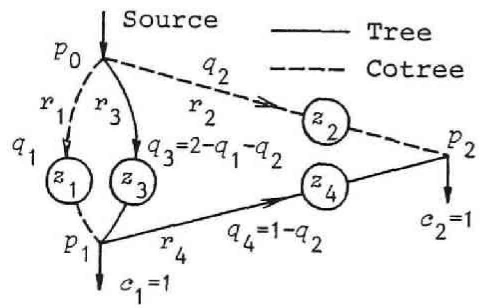


Fig. 5.1. Diagram of Network-1.

Second, if neither a pump nor a valve can be placed on some links, modify the objective function, Eq. (5.5), as

$$\text{minimize}\quad f' = \Sigma_i\ \{(q_iz_i+|q_iz_i|)/2+w_i|z_i|\} \tag{5.5'}$$

and assign a relatively large positive value to w_i of the link where placement of a pump and a valve is prohibited (hereafter, we call such a link a prohibited link), while zero to w_i of the link where the placement is allowed (an allowed link). By so doing, because use of a pump or a valve on a prohibited link needs large cost, they are not used as a necessary consequence. If the node-head condition can not be satisfied only by the pumps and the valves on the allowed links, some of those on the prohibited links should be put in use.

In addition to the present problem, the solution method proposed in the following is also effective for such a problem as to minimize the amount of water leakage from the network, under the assumption that the amount of leakage from each node is proportional to the effective head at the node. The optimal location of control points for one objective function differs from that for the other. And since managers and/or operators of waterworks have a variety of notions about the objective function in water distribution control, such a method that gives fundamental information about the location of pumps and valves in a flexible manner is quite useful.

5.3 Basic Algorithm (Algorithm 1)

(P1) is obviously a nonlinear programming problem. However, if Q_c is fixed to some value, and if z_i is written as

$$z_i = x_i - y_i \quad (x_i,\ y_i \geqq 0) \tag{5.8}$$

(P1) is reduced to a linear programming problem (LPP) whose unknowns are $X=(x_i)$ and $Y=(y_i)$. Let us denote the optimal value of the LPP with Q_c fixed by $f^*(Q_c)$ and the optimal point by (X^*, Y^*). Then the gradient of $f^*(Q_c)$ with respect to Q_c can be calculated by using shadow prices of LPP as follows:

$$\begin{aligned}\nabla f^*(Q_c) &= \frac{\partial f^*}{\partial Q_c} + \sum_{\mu\in S_L} \frac{\partial f^*}{\partial \underline{p}_\mu}\frac{\partial p_\mu}{\partial Q_c} + \sum_{\mu\in S_U} \frac{\partial f^*}{\partial \overline{p}_\mu}\frac{\partial p_\mu}{\partial Q_c} + \sum_{\phi} \frac{\partial f^*}{\partial l_\phi}\frac{\partial l_\phi}{\partial Q_c} \\ &= \frac{\partial f^*(X^*, Y^*, Q_c)}{\partial Q_c} + \begin{Bmatrix} \partial P_j(X^*, Y^*, Q_c)/\partial Q_c{}^T \\ \partial P_j(X^*, Y^*, Q_c)/\partial Q_c{}^T \\ \partial L(X^*, Y^*, Q_c)/\partial Q_c{}^T \end{Bmatrix}^T \Lambda \end{aligned} \tag{5.9}$$

T : transposition of a vector/matrix

S_L and S_U denote the set of active lower node-head constraints and that of active higher node-head constraints at (X^*, Y^*), respectively. Λ is the vector of the shadow prices at (X^*, Y^*) and its size is equal to $m+n-s$, i.e., the number of constraints of Eqs. (5.6) and (5.7).

Suppose that $\nabla f^*(Q_c)$ can be always defined. If $\nabla f^*(Q_c) \neq 0$, then there is a positive number δ (step size) which satisfies

$$f^*(Q_c - \delta \nabla f^*(Q_c)) < f^*(Q_c) \tag{5.10}$$

Hence, the optimal solution of (P1) can be found by iterating solutions of LPP and computations of the gradient, Eq. (5.9).

Since a pump or a valve can be located on every link, the head at every node can be set arbitrarily for any Q_c. Hence, LPP is always solvable.

It is a difficult problem how to determine a proper step size δ. One dimensional search is far from efficient because of the time needed for computing $f^*(Q_c)$. In fact, its computation is just a solution of an LPP. Then, the following algorithm is adopted.

<u>Basic algorithm (Algorithm 1)</u>.

Suppose that Q_c^0 and $\delta^0 > 0$ are given.

<u>Step 1</u> Set $k=0$.

<u>Step 2</u> Set $Q_c^k = Q_c^0$ and set $\delta^k = \delta^0$.

<u>Step 3</u> Compute $Q_c^{k'} = Q_c^k - \delta^k \nabla f^*(Q_c^k) / |\nabla f^*(Q_c^k)|$.

$|\cdot|$: Euclid norm of a vector

<u>Step 4</u> If $f^*(Q_c^{k'}) < f^*(Q_c^k)$, go to Step 5;

otherwise, go to Step 7.

<u>Step 5</u> Set $Q_c^{k+1} = Q_c^{k'}$, and set $\delta^{k+1} = \begin{cases} 1.5\delta^k & (\text{if } \delta^k \geqq \delta^{k-1} \geqq \delta^{k-2}) \\ \delta^k & (\text{except the above}) \end{cases}$

Step 6 Set $k=k+1$, and go to Step 3.

Step 7 Set $\delta^k=\delta^k/2$.

Step 8 If $\delta^k<\varepsilon$ stop; otherwise, return to Step 3.

ε: a reference small positive quantity for stopping the algorithm.

5.4 V-shaped Ravine

Algorithm 1 in the previous section is an extention of that of Alperovits et al. utilized for minimizing the total pipe cost in the design of WDN /5.4/. Unfortunately, the algorithm stops on the subspace of Q_c-space where $\nabla f^*(Q_c)$ can not be defined. Let us call such a subspace a "V-shaped ravine".

By way of example, in Network-1 of Fig. 5.1, suppose that $r_i=r=10^{1.85}$ and $p_0=55$m. The node-head condition is that the heads at demand nodes be more than or equal to 20m. The map of Fig. 5.2 shows directions of the negative gradient $-\nabla f^*(Q_c)$ on the q_1-q_2 plane of Network-1. This is obtained by assuming that $z_i=x_i\geq 0$ and $q_i x_i\geq 0$, i.e., without use of valves. On the two sides of the straight line $q_2=2-2q_1$, i.e., $q_3=q_1$, the negative-gradient directions are different drastically. That is, the line $q_3=q_1$ gives the bottom of a sharp ravine of the cost, a V-shaped ravine. Therefore, if the cost minimization by Algorithm 1 is started from a point on the ravine, say the point s indicated by ○, the flow pattern does not change at all; any small movement from s in the direction of $-\nabla f^*(Q_c)$ produces a higher cost. Then the optimal point o indicated by ◉ can not be reached. If the valve variables are introduced, the topography of the ravine is moderated somehow, but still it gives

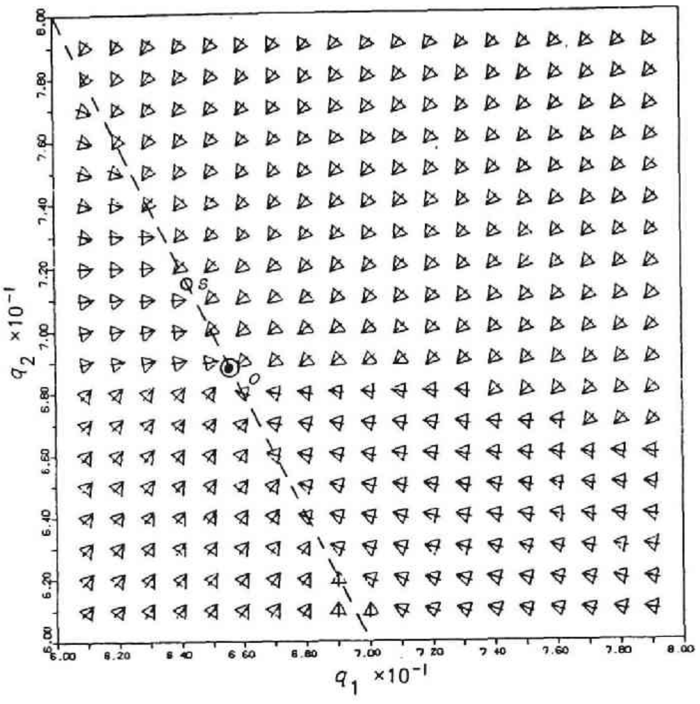


Fig. 5.2. Map of negative-gradient direction of f^* in Q_c-space of Network-1.

difficulty to Algorithm 1.

The V-shaped ravine is caused by the nondifferentiability of the objective function f: f is nondifferentiable with respect to q_i and z_i at $q_i=0$ and $z_i=0$, respectively. In fact, if the sign of $q_i z_i$ switches, the objective function of LPP changes, and consequently the V-shaped ravine is formed. It is to be noted that the sign of z_i (z_i^*) can not be known until LPP is solved.

The subspace of $q_i=0$ is a hyperplane in Q_c-space, because q_i is a linear function of the components of Q_c. $z_i=0$ is the subspace where the basis (the set of basic variables) of LPP changes, i.e., at least one of x_i^* and y_i^* switches between zero and positive.

The V-shaped ravine can also be explained through Eq. (5.9). The first term of Eq. (5.9) is discontinuous at the subspace of q_i=0 and z_i^*=0. The second term is also discontinuous at the subspace of z_i^*=0, because some components of Λ changes discontinuously there according to the change of the basis of LPP. (Note: Therefore, the V-shaped ravine can emerge even if the objective function is smooth.)

Now, let us consider the subspace of z_i^*=0 in detail. It should be noted that we use the linear graph Ξ where inflows from sources and outflows from consumption nodes are represented by the flow rates of the in/outflow links connecting to the datum node. Figure 5.3 shows Ξ of Network-1. Consider the neighborhood of $\overline{Q}_c$, a point on a V-shaped ravine. First, if neither the upper constraint nor the lower constraint on p_μ is active at $\overline{Q}_c$, the constraints are also not active in the proper neighborhood of $\overline{Q}_c$, hence the consumption link μ is not concerned with the change of basis of LPP. Second, if x_i^*>0 holds for pipe link i, x_i^* remains positive for a small change in q_i and also for a small change in h_i caused by small changes in the flow rates in other links, so far as such changes are in the proper neighborhood of $\overline{Q}_c$. The same is valid when y_i^*>0 holds for pipe link i. Thus, change of the basis is possible

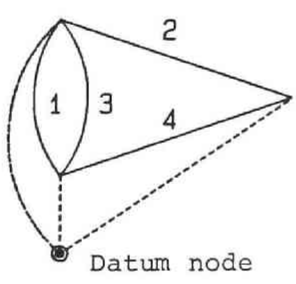


Fig. 5.3. Ξ of Network-1.

only on the subgraph Ξ_b obtained by deleting the above mentioned consumption links and pipe links from Ξ.

Consider consumption link μ in Ξ_b. Since its node head is constrained to the lowest or the highest allowable value, the node-head condition is equivalent to HDLL. By way of example, in Network-1, HDLL of the loop of links 1 and 3, and the node-head condition at node 1 (assumed to be active) are written as follows:

$$x_1^*-y_1^* - (x_3^*-y_3^*) = r_1q_1|q_1|^{0.85} - r_3(2-q_1-q_2)|2-q_1-q_2|^{0.85}$$
$$x_1^*-y_1^* = \underline{p}_1-p_0+r_1q_1|q_1|^{0.85}$$

where the components of X and Y are collected on the left sides of the equations, and the components of Q_c and constants on the right sides.

Find out a full set of independent loops in Ξ_b and write down HDLL of those loops. Let us denote the set of the right sides of those equations by $G(Q_c)=(g_\kappa(Q_c))$, which are called loop head-loss terms from their characters.

Now, since the left sides of the equations are all zero,

$$G(Q_c) = 0 \tag{5.11}$$

is satisfied at $\overline{Q}_c$. If some $g_\kappa(Q_c)$ becomes positive or negative, at least one of $z_i^*=x_i^*-y_i^*$ switches its sign. That is to say, Eq. (5.11) describes the V-shaped ravine.

If Q_c, i.e., the flow pattern, is changed along the V-shaped ravine, a new descent of $f^*(Q_c)$ becomes possible. The descent along the ravine does not put any pump nor valve in all pipe links in Ξ_b.

5.5 Revised Algorithm

Based on the foregoing discussion, an algorithm to descend along the bottom of a ravine is constructed in this section.

5.5.1 Algorithm for the search of ravine equations (Algorithm 2)

Consider the k-th iteration of the basic algorithm. Let $\mathrm{LP}(Q_c)$ denote the LPP at Q_c, and let δ_r be a small positive value for judging an encounter with a V-shaped ravine.

Step 0 If $f^*(Q_c^{k'})>f^*(Q_c^k)$, for $Q_c^{k'}=Q_c^k-\delta^k\nabla f^*(Q_c^k)/|\nabla f^*(Q_c^k)|$ with $\delta^k<\delta_r$, that is, if the cost can not be improved even if the step size δ^k is made sufficiently small, go to Step 1; otherwise, iterate the basic algorithm.

Step 1 Find out the subgraph Ξ_b based upon the solutions $X^k=(x_i)$ and $Y^k=(y_i)$ of $\mathrm{LP}(Q_c^k)$, and the solutions $X^{k'}=(x_i')$ and $Y^{k'}=(y_i')$ of $\mathrm{LP}(Q_c^{k'})$.

1) Let all the source links be included in Ξ_b.

2) Consider pipe link i. If $(x_i-y_i)(x_i^*-y_i^*)>0$, since link i is not concerned with change of the basis, let link $i\notin\Xi_b$; otherwise, let link $i\in\Xi_b$.

3) Consider consumption link μ. If $\underline{p}_\mu<p_\mu^k<\overline{p}_\mu$ and $\underline{p}_\mu<p_\mu^{k'}<\overline{p}_\mu$, since the constraints are not active both at Q_c^k and at $Q_c^{k'}$, let link $\mu\notin\Xi_b$; otherwise, let link $\mu\in\Xi_b$. Here, p_μ^k and $p_\mu^{k'}$ denote the heads at node μ in the solutions of $\mathrm{LP}(Q_c^k)$ and $\mathrm{LP}(Q_c^{k'})$, respectively.

Step 2 In Ξ_b, find out a full set of independent loops by spanning a tree, and construct their loop head-loss terms $g_\kappa(Q_c)$ $(\kappa=1, 2, \ldots, \tau_0)$.

Step 3 If the sign of q_i at $Q_c^{k'}$ differs from that at Q_c^k, and if the equation of $q_i=0$ is linearly independent to the set of $\{g_\kappa(Q_c)=0\ (\kappa=1, 2, \ldots, \tau_0)\}$, the equation of $q_i=0$ is also added to ravine equations as $g_\kappa(Q_c)=0$, κ running from τ_0+1 to τ.

5.5.2 Algorithm for descent along a V-shaped ravine (Algorithm 3)

Suppose that the flow pattern is now Q_c^k and is close to a V-shaped ravine. Further, suppose that, by Algorithm 2, the ravine equations are turned out to be

$$G(Q_c) \triangleq (g_\kappa(Q_c)) = 0 \quad (\kappa = 1, 2, \ldots, \tau) \tag{5.12}$$

Step 1 Let v be the projected vector of $-\nabla f^*(Q_c)$ on the tangential hyperplane of the V-shaped ravine of Eq. (5.12). Change the flow pattern to $Q_c^{k'}$ which is in the direction of v by a step size δ^k:

$$Q_c^{k'} = Q_c^k - \delta^k v/|v| \tag{5.13}$$

where

$$v = (I-D^T(DD^T)^{-1}D)(-\nabla f^*(Q_c)), \quad D \triangleq (\partial G^T(Q_c)/\partial Q_c)^T|_{Q_c=Q_c^k}$$

Step 2 By use of the Newton-Raphson method, change the flow pattern from $Q_c^{k'}$ to $Q_c^{k''}$ which satisfies Eq. (5.12) as shown in Fig. 5.4.

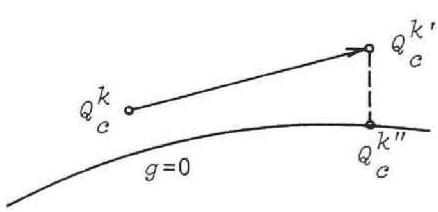


Fig. 5.4. Descent along a V-shaped ravine.

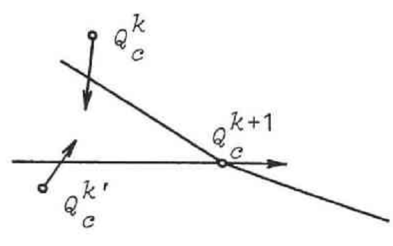


Fig. 5.5. Intersection of V-shaped ravines.

The equation of the hyperplane which passes $Q_c^{k'}$ and orthogonal to the tangential hyperplane of Eq. (5.12) is given by

$$Q_c - Q_c^{k'} = (D')^T t, \quad D' \triangleq (\partial G^T(Q_c)/\partial Q_c)^T |_{Q_c = Q_c^{k'}} \tag{5.14}$$

where $t = (t_1, t_2, \ldots, t_\tau)^T$ is the vector of arbitrary real numbers. Equation (5.12) is solved simultaneously along with Eq. (5.14). Equation (5.12) can be expanded around $Q_c^{k'}$ as:

$$G(Q_c^{k'}) + D'(Q_c - Q_c^{k'}) = 0 \tag{5.15}$$

Then, from Eqs. (5.14) and (5.15), we can obtain

$$D'(D')^T t = -G(Q_c^{k'}) \tag{5.16}$$

Hence, by iterating the solution of Eq. (5.16) with respect to t and the substitution such as

$$Q_c^{k'} \leftarrow Q_c^{k'} + D' t \tag{5.17}$$

if $|t_\kappa| \cong 0$ ($\kappa = 1, 2, \ldots, \tau$) is satisfied, we obtain $Q_c^{k''}$ by the substitution

$$Q_c^{k''} \leftarrow Q_c^{k'}$$

It must be noted that more than one ravine may be found out at a time by Algorithm 2 as shown in Fig. 5.5. In such a case, Eq. (5.12) denotes the intersection of those ravines. In general, the ravines terminate at an intersection and a new ravine starts there. (Note: If the intersection is a point, it may be the optimal point.) Then the nearest point on the intersection from Q_c^k is chosen as Q_c^{k+1}, and descent is restarted from Q_c^{k+1} by using Algorithm 1. The following step is appended for this purpose.

Step 3 If $f^*(Q_c^k) < f^*(Q_c^{k''})$, then execute Step 1 and Step 2 with a smaller step size, $0.1\delta^k$. If f^* does not decrease even with this step size, find the nearest point on the subspace of Eq. (5.12) from Q_c^k and let the point be Q_c^{k+1}.

5.5.3 Revised algorithm

Algorithms 1 through 3 are combined as follows.

1) Start minimization by using Algorithm 1, the basic algorithm.

2) When the cost can not be improved even if step size δ^k is made smaller than δ_r as in the Step 0 of Algorithm 2, switch into Algorithm 2 and find ravine equations.

3) The descent along the ravine bottom is made by use of Algorithm 3, and this descent is iterated just as in Algorithm 1.

4) The descent along the ravine bottom may be stopped at some point by an encounter with another V-shaped ravine. If it is stopped, restart minimization by using Algorithm 1 from that point. This is because equations of the present ravine are not necessarily included in the set of equations of a new ravine.

As to δ_r for judging encounter with a ravine, since the step size δ^k becomes smaller and smaller as Q_c^k approaches to the optimal point, we halve δ_r at every application of Algorithm 2, considering the balance between δ^k and δ_r.

Finally, the procedure is stopped when both

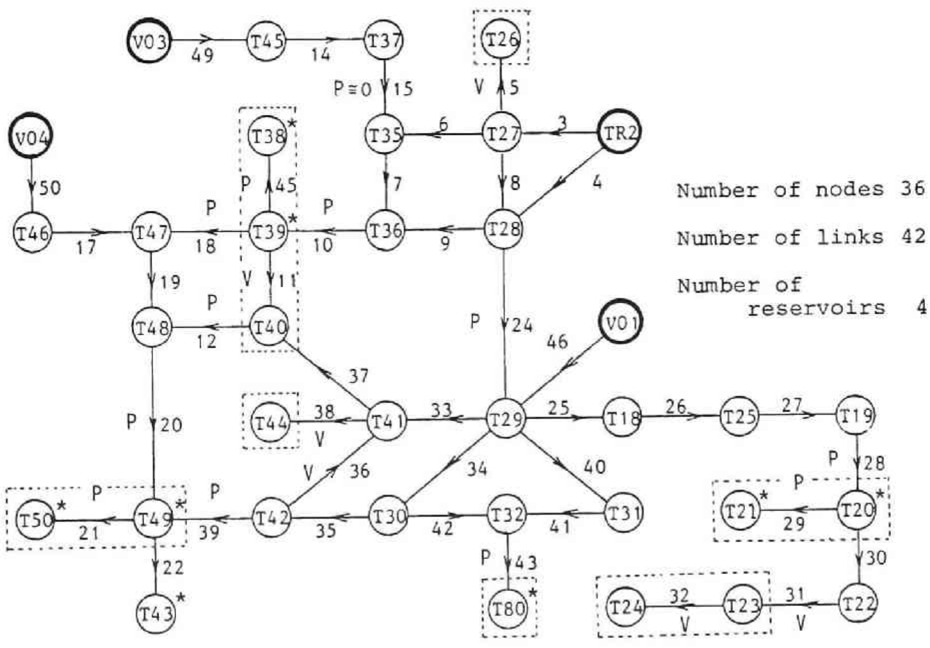
$$\delta^k < \varepsilon \quad \text{and} \quad \delta_r < \varepsilon$$

hold.

The proposed algorithm has similarity to those for solving minimax problems in coping with discontinuity of the gradient. However, as only one function $f^*(Q_c)$ is considered in our problem, the techniques for minimax problems /5.5/ where more than one functions are involved are not directly applicable. Moreover, since evaluation of f^* is much time consuming (it needs an LP solution), reduction of the function evaluation frequency must be a key point of the algorithm. The principal feature of the present algorithm is summarized as follows: It tries to find out the subspace where the gradient is discontinuous and makes descent just along that subspace.

5.6 Example

The basic algorithm and the revised algorithm are applied to some networks of practical size. Here, only the results of the application to Network-2 of Fig. 5.6 is explained.



Note 1) An arrow on each link denotes the direction of the flow.

Note 2) P and V denote a pump and a valve respectively.

Note 3) The heads at the demand nodes which are circled by dotted lines are the lowest admissible value.

Fig. 5.6. Diagram of Network-2 (Result of the optimazation).

Network-2 is a model of a real WDN.

The node-head condition is that the effective head must be kept between 20m and 40m at every node. Without use of any pumps and valves, the head is lower than 20m at the nodes indicated by * in Fig. 5.6, but is not higher than 40m at any node (the maximum value is only 26.7m).

For cost minimization, the initial step size δ^0, the initial value of δ_r and ε for stopping the algorithm are set as follows:

$$\delta^0 = 10^{-3}, \quad \varepsilon = 10^{-5}, \quad \delta_r = 10^{-5}$$

To begin with, consider the case that a pump and a valve can be used on every link. The minimizations by use of the basic algorithm and the revised algorithm are started from two kinds of initial flow pattern:

Case 1 The steady-state flow pattern without use of any pumps and valves.

Case 2 Flow rates in the cotree links are equal to $0.1\text{m}^3/\text{s}$.

Main results of the above four cases are shown in Table 5.2. The final cost by use of the basic algorithm depends much on the initial flow pattern, and are relatively high being

Table 5.2. Results of the minimizations

Algorithm	Initial pattern	Number of LP solutions	Cpu time [s] Total	Cpu time [s] LP	Final cost	Initial cost
Basic algorithm	Case 1	28	25.8	22.8	0.9364	0.951
	Case 2	123	134.7	121.3	0.9728	75.0
Revised algorithm	Case 1	76	67.6	62.3	0.8906	0.951
	Case 2	102	90.5	83.4	0.8908	75.0

compared with that by use of the revised algorithm. These results mean that the iteration stops at some V-shaped ravine.

Contrary to this, the final cost by the revised algorithm is independent of the initial flow pattern. Hence it can be concluded that the revised algorithm works well for descent along the V-shaped ravine. In addition, it is noted that the cpu time necessary for search of ravine equations is quite small compared to that for LP.

The following are some comments on the result obtained by the revised algorithm. Figure 5.6 shows selected locations of pumps and valves. Some valves such as on links 5, 31, 32 and 38 can be removed without affecting the cost for pumping. These valves are just for setting the heads of the downstream nodes to 20m, and hence even if they are removed, the heads do not exceed 40m. As is shown in this example, though there are generally an infinite number of optimal solutions of X and Y, a unique value of the minimum cost is obtained.

In our formulation, all links are candidates for putting pumps or valves, but their usage is fairly limited in the result. A typical finding is that the pumps should be installed in the downstream parts of the network in a decentralized manner for saving of the operating cost.

Further, the obtained location of pumps and valves gives us much insight into characteristics of the network; pipes which should be enlarged, inadequate loop formation which destroys the balance among supplies from several reservoirs, and so on.

Next, consider to use only the pumps on links 10, 20, 21, 24 and 43. The heads of those pumps are given large values in the preceding case. The usage of valve is not prohibited on every link. For that, the term $w_i|z_i|$ in Eq. (5.5)' is replaced by $w_i|x_i|$ (if $q_ix_i>0$) or $w_i|y_i|$ (if $q_iy_i>0$), and the value of w_i of the prohibited links is set as 10^5.

In the result, in addition to five pumps on the allowed links, a pump of 0.44m is used also on link 46. Without this pump, the node-head condition can not be satisfied. Since a pump is used at the exit of a source, the heads take relatively large values all over the network, and then the cost goes up to 2.3274 (the value of $\Sigma(q_iz_i+|q_iz_i|)/2$). The valves are used on four links.

5.7 Concluding Remarks

In this chapter, a way for formulating the uniform pressure control problem of WDN, and an efficient algorithm for solving it have been proposed. That is, the problem is formulated in a nondifferentiable optimization problem, and it is solved by iterative solutions of linear programs with a special consideration on V-shaped ravines.

The algorithm is applied to some networks of practical size, and it is confirmed that the unique minimum-cost solution can be obtained. The minimum-cost solution gives us much knowledge about the characteristics of the network that can not be given by other methods.

Extensions of the present study such as consideration of the cost for water leakage, pump and/or valve installation and valve operation are possible.

CHAPTER 6 GENERAL CONCLUSION

This text has presented some studies on analysis, design and control of the water distribution network (WDN).

In chapter 2, the fundamentals of mathematical models of WDN have been discussed in refined and generalized manners, so that they can effectively be used for formulating a variety of problems of WDN. The models of hydraulic elements of intelligent type, which are currently getting popular because of their excellent operating performance, have also been introduced.

In Chapter 3, a comprehensive study on the steady-state flow analysis (SSFA) has been developed with the aims of (1) generalizing the SSFA so that it allows more freedom in choosing quantities to be prescribed and to be calculated than the traditional (basic) SSFA, and (2) establishing the most efficient method for fast solution of the SSFA. Then it has been concluded that the Mesh-Flow method, linking the formulation by the mesh-flow method and the solution by the original Newton-Raphson (NR) method, is most efficient for the basic SSFA, while the linkage of the node-equation formulation and NR-B method is most recommendable for the generalized SSFA.

The generalized SSFA has been shown quite effective to cope with a wide class of problems. It has also been shown that the solution of the formulated problem is not unique in some cases. Therefore, a more investigation is needed on how to attain efficiently the most desirable solution in such cases.

A method of formulation of the generalized SSFA which corresponds to the mesh-flow method of the basic SSFA is now under investigation.

In Chapter 4, new methods for modeling and design of the irrigation-water distribution network (IWDN) have been developed. The "link outflow model" reflects the salient feature of IWDN and makes a more precise analysis and design of IWDN possible than before. The proposed method of design establishes a concrete index for ensuring reliability and reduces the solution process to linear calculations. Those proposed methods have already been used for practical design of large-scale networks, and have proved their excellent capability compared with conventional techniques which need to solve nonlinear programs.

A layout of the network, which is supposed to be prescribed in our discussion, and an arrangement of the major links give crucial influence to the system reliability. It is now an open problem how to determine them. However, the proposed method of design will also facilitate the study of this problem.

Chapter 5 has proposed a way for formulating the uniform pressure control problem of WDN, and an efficient algorithm for solving it. They enable us to find out desirable location of pumps and valves as well as their operation schemes. The solution gives us much knowledge about the characteristics of the network that can not be given by other methods.

The presented problem is quite interesting from the viewpoint of nondifferentiable optimization. Hence, it is worthwhile to be examined in a manner mathematically more strict.

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